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Gravity-driven flow of Herschel-Bulkley fluid in a fracture and in a 2D porous medium

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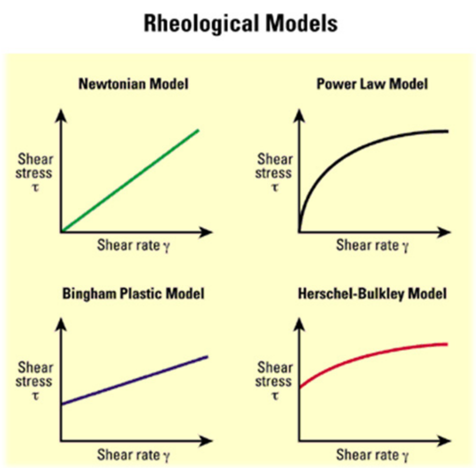
1. Implications of fluid rheology on flow in fractures and porous media

The behaviour of Herschel-Bulkley fluids flowing in a narrow channel (a fracture) has not been investigated to the same extent as flows in wide channels, and deserves an in-depth analysis due to the numerous practical applications of the process, such as polymer processing, heavy oil flow, gel cleanup in propped fractures, drilling processes.



A practical example: drilling fluids

There are several different types of drilling fluids, based on both their composition and use. The models of most interest in drilling fluid technology are the Bingham plastic, power law and Herschel-Bulkley (HB). In many cases (e.g., NAFs and clay-based WBMs) the HB equation is preferred to power law or Bingham relationships because it results in more accurate predictions of rheological behaviour at low shear rate.

Drilling fluid (mud): in mining engineering, it is used to aid the drilling of boreholes into the earth.

Darcy's law, valid for Newtonian fluids, has been extended, with various methodologies, to power-law non-Newtonian fluids and experimentally validated [1-2]. However, even though the power-law approximation provides an accurate interpretation of fluid behaviour in several flow conditions, it does not cover other classes of fluids exhibiting yield stress. These are better described by models such as Herschel-Bulkley three parameters [3], Cross (four parameters, [4]), and Carreau-Yasuda (four or five parameters, [5-6]).

Present work

A theoretical model and its experimental validation for 2D flows of a Herschel-Bulkley fluid in a narrow fracture and in a porous medium is presented. The theoretical model is general, while computations and experiments refer mainly to a specific situation (the injected volume quadratic in time) where a simple self-similar solution is available. An expansion method has been applied to handle, with some restrictions, the general case of an injected volume which is power-law over time; the general method has likewise been experimentally validated.

2. Model description for free-surface flow in a narrow fracture

HB model for a shear thinning/thickening fluid with yield stress:

$$\begin{cases} \tau = \left(\mu_0 \dot{\gamma}^{n-1} + \tau_p \dot{\gamma}^{-1}\right)\dot{\gamma}, & \tau \geq \tau_p \\ \dot{\gamma} = 0, & \tau < \tau_p \end{cases}$$

τ = stress;
$\dot{\gamma}$ = strain rate.

μ_0 is the consistency index and represents a viscosity-like parameter;
τ_p is the yield stress of the fluid;
n is the fluid behavior index.

$n < 1$ shear-thinning
$n = 1$ Bingam
$n > 1$ shear-thickening

For free-surface flow through a narrow fracture of width Ly:

The velocity $y(x, y)$ in the x-direction must satisfy

$$\begin{cases} \tau = \left(\mu_0 \left|\frac{\partial u}{\partial y}\right|^{n-1} + \tau_p \left|\frac{\partial u}{\partial y}\right|^{n-1}\right)\frac{\partial u}{\partial y}, & \tau_{xy} \geq \tau_p \\ \dot{\gamma} = 0, & \tau_{xy} < \tau_p \end{cases}$$

where τ_{xy} represents the cross gap stress and y is the cross gap direction.

continuity of mass:

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial t}(\bar{u}h)$$

volume of fluid in the GC:

$$L_y \int_0^\infty h(x,t)dx = Qt^\alpha$$

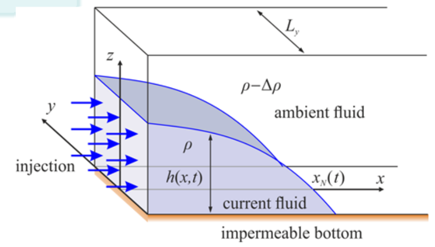


Figure. Diagram showing the setup of axes and fluid orientation in flow through a narrow fracture (Hele-Shaw cell).

Assuming a zero slip velocity, the evolution equation for $h(x,t)$ is:

$$\frac{\partial h}{\partial t} = \mathrm{sgn}\left(\frac{\partial h}{\partial x}\right)\Omega\frac{\partial}{\partial x}\left[h\left|\frac{\partial h}{\partial x}\right|^{\frac{1}{n}}\left(1-\kappa\left|\frac{\partial h}{\partial x}\right|^{-1}\right)^{\frac{n+1}{n}}\left(1+\left(\frac{n}{n+1}\right)\kappa\left|\frac{\partial h}{\partial x}\right|^{-1}\right)\right]$$

where $\kappa = 2\tau_p/(\Delta\rho g L_y)$ is a non dimensional number (ratio between the yield stress and the gravity related stress) and Ω is a velocity scale.

3. Self-similar solution

For $\alpha = 2$, a velocity scale given by $(Q/L_y)^{1/2}$ arises, and a self-similar solution of the form $h = tf(\eta)$, with $\eta = x/t$, yields:

$$f - \eta f' = -\delta\left[f|f'|^{\frac{1}{n}}\left(1-\frac{\kappa}{|f'|}\right)^{\frac{n+1}{n}}\left(1+\left(\frac{n}{n+1}\right)\frac{\kappa}{|f'|}\right)\right]',$$

$$\int_0^{\eta_e} f(\eta)\,\mathrm{d}\eta = 1.$$

where $\delta = (L_y\Omega/Q)^{1/2}$ is the ratio between the two velocity scales.
This system admits a simple solution, namely a linear profile for $f(\eta)$ [7]. Supposing a solution in the form $f(\eta) = A(\eta_e - \eta)$, for some constant $A > 0$ and $\eta_e > 0$, it is possible to obtain:

$$A\eta_e = \delta A^{(n+1)/n}\left(1-\frac{\kappa}{A}\right)^{(n+1)/n}\left[1+\left(\frac{n}{n+1}\right)\frac{\kappa}{A}\right], \quad \eta_e = \sqrt{\frac{2}{A}}.$$

Eliminating η_e gives one non linear equation to solve for A.

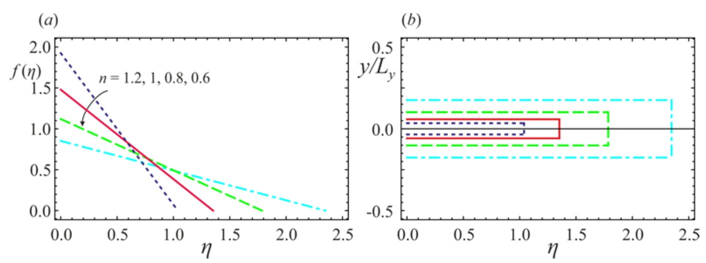


Figure. (a) Shape of the similarity solution in a Hele-Shaw cell ($\alpha = 2$) for different values of n; (b) plug regions.

2D flow in a porous medium

The case of flow through a porous medium requires the formulation of the equivalent Darcy's law for a HB fluid, which may be written:

$$d\nabla p = \chi\tau_p + \beta\mu_0\left(\frac{\bar{u}}{d}\right)^n$$

where d is the diameter of the grains, ∇p the pressure gradient, τ_p the yield stress, n the flow behavior index, $\bar{u}$ the darcian velocity and χ and β are coefficients. Under the relevant shallow water approximation and under the constraint $|\partial h/\partial x| > \kappa_p$, the evolution equation is

$$\frac{\partial h}{\partial t} = -\delta_p\frac{\partial}{\partial x}\left[h\left|\frac{\partial h}{\partial x}\right|^{\frac{1}{n}}\left(1-\kappa_p\left|\frac{\partial h}{\partial x}\right|^{-1}\right)^{\frac{1}{n}}\right]$$

The self-similar solution $h = tf(\eta)$ and $f(\eta) = A_p(\eta_{ep} - \eta)$ lead to

$$\frac{2}{A_p} = {\delta_p}^2(A_p - \kappa_p)^{2/n}, A_p{\eta_{ep}}^2 = 2.$$

For given values of κ_p and δ_p, it is possible to solve the first equation numerically in the unknown A_p an then to compute η_{ep}.

Asymptotic analysis for $\alpha \neq 2$

We propose the following expansion:

$$h = \eta_N^{n+1}t^{F_2}\left[f_0(\zeta) + \sigma f_1(\zeta) + \sigma^2 f_2(\zeta) + \ldots\right],$$

$$x = \eta t^{F_1}\left(1 + \sigma X_1 + \sigma^2 X_2 + \ldots\right),$$

in the regime $\sigma \equiv \kappa t^{n(2-\alpha)/(n+2)} \ll 1$.
X_1, X_2, … are constant to be evaluated; $f_0(\zeta)$ and η_N are given by the similarity solution for power low fluids ($\kappa \to 0$):

$$h = \eta_N^{n+2}t^{F_2}f(\zeta), \ \eta = xt^{-F_1}, \ \eta_N = \left(\int_0^1 f\mathrm{d}\zeta\right)^{-1/(n+2)}, \zeta = \eta/\eta_N$$

The evolution equation becomes:

$$\left(f_1|f_0'|^{1/n} - \frac{1}{n}f_0|f_0'|^{1/n-1}f_1'\right)' + F_2 f_1 - F_1\zeta f_1' = \frac{1}{\eta_N^n}\frac{2n+1}{n(n+1)}\left(f_0|f_0'|^{1/n-1}\right)'$$

which is inhomogeneous linear ODE for the unknown function f_1, with a forcing term modulated by the fundamental solution f_0.
The numerical integration of this equation requires two boundary conditions for $\zeta \to 0$, obtained again by expanding in series near the front of the current.

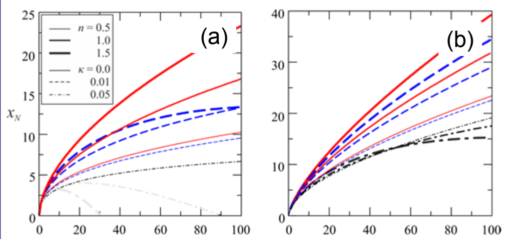


Figure. The effects of the 1st order correction: (a) $\alpha = 0.5$, (b) $\alpha = 1$.

4. The experiments

In order to validate the theoretical model, two series of experiments were conducted (i) in a Hele-Shaw cell with a small gap, simulating a fracture, and (ii) in the same cell with a larger gap and filled with glass beads of uniform size, reproducing a 2D porous medium.

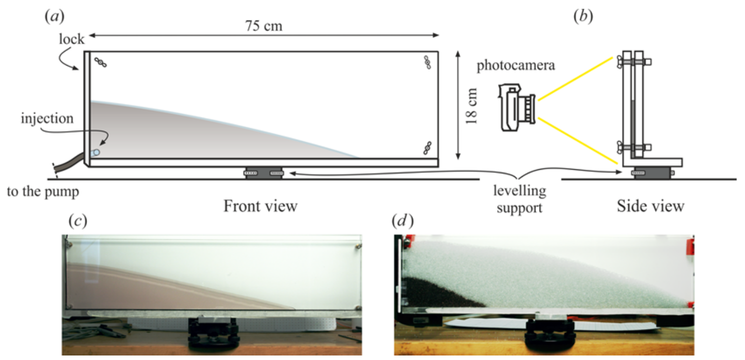


Figure. A sketch of the experimental rectangular channel. (a) Front view, (b) side view, (c) a snapshot of the channel during Exp. B1 (the shaded area is the advancing current), and (d) a snapshot of the channel filled with glass beads during Exp. A1 (the dark area is the advancing current, the grey area is the porous medium not yet reached by the current).

Rheometry of the fluid

Most measurements were conducted with a parallel plate rheometer (DSR Anton Paar Physica MCR 101), and with a strain-controlled rheometer (coaxial cylinders Haake RT 10 RotoVisco). In order to limit the slip, the surfaces of the cup and of the rotor were roughened with strips of Sellotape, the surface of the plates were roughened with sand paper P-60 glued on the smooth surface.

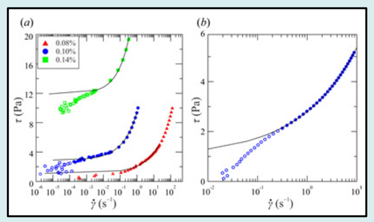


Figure. (a) Experimental shear-stress shear-rate curves for three fluids Carbopol 980 0.08; 0.10; 0.14%. Measurements with the coaxial cylinders rheometer. (b) Experimental shear-stress shear-rate curves for Carbopol 980 0.10%. Measurements with the plane plate rheometer.

Comparison of theory with experiments

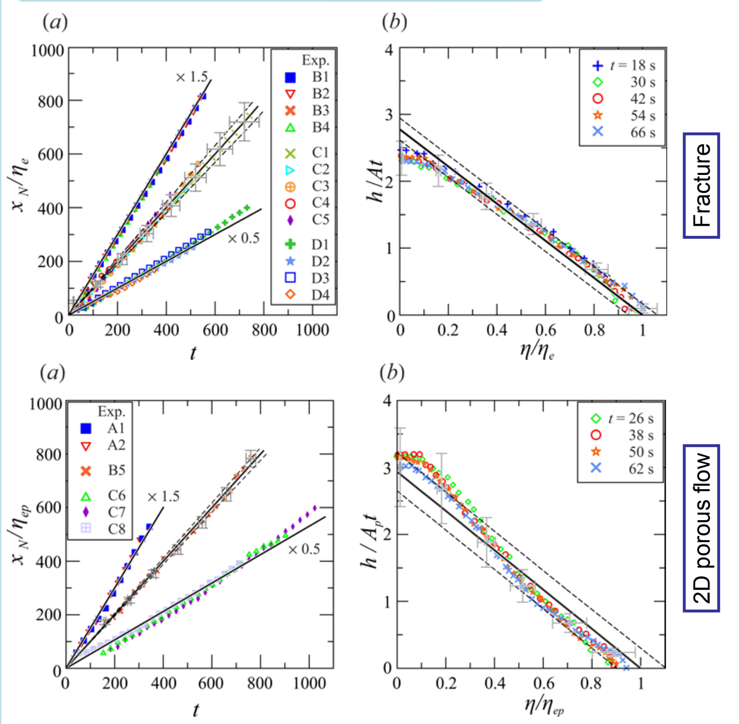


Figure. The graphs show (a) the front position x_N/η_e as a function of dimensionless time t for all tests. The three bold lines represent the perfect agreement with theory for the three different fluids used in the experiments; (b) the dimensionless profile of the current at different times for Exp. B3.

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