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H41E-1368. NON-NEWTONIAN FLUIDS IN SUBSURFACE ENVIRONMENTS: RHEOLOGY, MODELING, AND APPLICATIONS

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Modeling of non-Newtonian Free-Surface and Confined Flow in Porous Media: a Review of Theoretical and Experimental Results for Power-Law Fluids

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ABSTRACT

Several environmental contaminants and remediation agents exhibit rheological complexity. Crude oil and displacing agents in EOR operations are rheologically nonlinear. These applications prompt the need for a theoretical analysis of non-Newtonian flow in natural porous and fractured media, considering gravity-driven and confined flows, different geometries and diverse boundary conditions. We present a review of the results obtained by our group concerning the modeling of power-law fluids, as this constitutive law is amenable to self-similar solutions which may act as benchmarks even for more complex rheology. First, closed form results were obtained for gravity currents advancing in plane or cylindrical geometry, deriving scalings for current length and thickness. Analogous results were obtained for confined flows in various geometries; here, scalings were obtained for pressure front position and pressure field.
Based on these benchmarks, the analytical models were refined introducing two additional factors: medium heterogeneity and topographic control. The inherent heterogeneity of natural media was modeled within a simplified framework considering continuous variations of spatial properties. Topographic control was introduced considering flows in porous channels of different shapes. Both factors proved relevant for the spreading of gravity currents as they influence the extent and shape of porous domain invaded by the contaminant, or reached by the remediation agent.
Our theoretical results were validated against multiple sets of experiments, conducted with different combinations of spreading scenarios and types of heterogeneity or channelization. Two basic experimental setups were employed, adopting either reconstructed porous media made of glass beads, or Hele-Shaw analogues. To this end, existing Hele-Shaw analogies for porous flow of power-law fluids were extended to heterogeneous media. All scalings derived for the current front and thickness were confirmed by our experiments, with an agreement between theory and experiments improving with time. A comparison between the key exponents governing the propagation of current or pressure front allows to determine the relative influence of rheology, heterogeneity, and domain shape and geometry.

THEORETICAL BACKGROUND

- Power-law fluid constitutive equation in simple shear
m consistency index, n flow behavior index

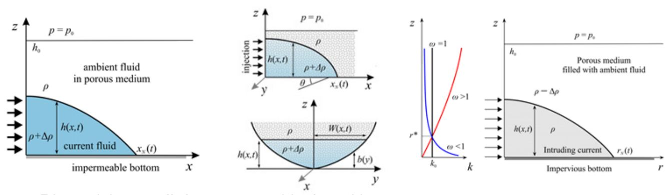
$$\tau = m\dot{\gamma}\left|\dot{\gamma}\right|^{n-1}$$

- Darcy law for flow in p.m.
u Darcy velocity
p pressure
k, ϕ permeability, porosity

$$\nabla p - \rho\mathbf{g} = -\frac{1}{\Lambda k^{(1+n)/2}}\left|\mathbf{u}\right|^{n-1}\mathbf{u},$$

$$\Lambda = \Lambda(\phi, m, n) = \frac{8^{(n+1)/2}}{2}\left(\frac{n}{3n+1}\right)^{n}\frac{\phi^{(n-1)/2}}{m}$$

Free-surface flow



- Plane (x) or radial geometry (r); time (t)
- Motion driven by density difference $\Delta\rho$ between heavy intruding fluid and light fluid saturating the medium; also channel slope if inclined
- Sharp interface
- Current height is thin compared to length and porous medium thickness
- Negligible surface tension effects
- Under previous assumptions, vertical velocities in the intruding fluid are neglected, the pressure within is hydrostatic, ambient fluid is taken to be at rest; the thickness h of the current is determined as $h(x,t)$, or $h(r,t)$
- Current volume introduced at the system boundary $V \propto t^{\alpha}$ ($\alpha = 0$ constant volume, $\alpha = 1$ constant flux injection) or constant first spatial moment Q (dipole flow)
- Zero height at the front $x_N(t)/r_N(t)$ is maximum extension of the current
- Heterogeneity along vertical z given by $k \propto z^{\omega-1}$ ($\omega = 1$ homogeneous)
- Heterogeneity along horizontal x,r given by $k \propto z^{\beta}$ ($\beta = 0$ homogeneous)
- Channel shape for plane geometry given by κ parameter ($\kappa \to \infty$, unbounded plane geometry)

Self-similar solutions govern the long-time evolution of the current. Scalings for the current extension and thickness are

$$X_N, R_N \approx T^{F_2} \quad H \approx T^{F_3}$$

Free surface flow

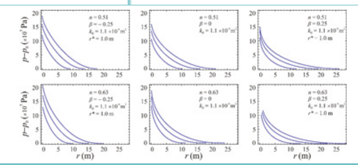
- Injected volume (all except dipole flow) $V \propto T^{\alpha}$
- First spatial moment (dipole flow) $Q = const$
- Velocity of the current $U_N \propto T^{F_2-1}$
- Maximum extension of the current $X_N, R_N \propto T^{F_2}$
- Thickness of the current $H \propto T^{F_3}$
- Aspect ratio and average gradient $\overline{H}/X_N, \overline{H}/X_N, \left(\partial\overline{H}/\partial X\right), \left(\partial\overline{H}/\partial R\right) \propto T^{F_3-F_2}$

	Plane unbounded [ref. 9]	Radial [ref. 8]	Plane channelized [ref. 3]	Plane, vertical k heterogeneity [ref. 1]	Radial, vertical k heterogeneity [ref. 5]	Dipole, vertical k heterogeneity [ref. 2]	Plane, horizontal k heterogeneity [ref. 1]	Radial, horizontal k heterogeneity [new result, 2016]
Parameters	α, n	α, n	α, n, κ κ = channel shape $\kappa \to \infty$, plane case	α, n, ω $k \propto z^{\omega-1}$ homogeneous if $\omega = 1$	α, n, ω $k \propto z^{\omega-1}$ homogeneous if $\omega = 1$	α, n, ω $k \propto z^{\omega-1}$ homogeneous if $\omega = 1$	α, n, β $k \propto x^{\beta}, r^{\beta}$ homogeneous if $\beta = 0$	α, n, β $k \propto x^{\beta}, r^{\beta}$ homogeneous if $\beta = 0$
$F_2 - 1$	$\frac{\alpha-2}{n+2}$	$\frac{\alpha-3}{n+3}$	$\frac{\alpha\kappa-(2\kappa+1)}{n+1+\kappa(n+2)}$	$\frac{\alpha[(n+1)(\omega-1)+2]-n(\omega-1)-(\omega+3)}{2(n+2)+(n+1)(\omega-1)}$	$\frac{\alpha[(n+1)(\omega-1)+2]-2n(\omega-1)-2(\omega+2)}{2(n+3)+2(n+1)(\omega-1)}$	$-\frac{n(\omega-1)+2+\omega}{2(n+3)+2(n+1)(\omega-1)}$	$\frac{2\alpha-[4-\beta(n+1)]}{2(n+2)-\beta(n+1)}$	$\frac{2\alpha-[6-\beta(n+1)]}{2(n+3)-\beta(n+1)}$
F_2	$\frac{\alpha+n}{n+2}$	$\frac{\alpha+n}{n+3}$	$\frac{\alpha\kappa+n(\kappa+1)}{n+1+\kappa(n+2)}$	$\frac{\alpha[(n+1)(\omega-1)+2]+2n}{2(n+2)+(n+1)(\omega-1)}$	$\frac{\alpha[(n+1)(\omega-1)+2]+2n}{2(n+3)+2(n+1)(\omega-1)}$	$\frac{n}{2(n+3)+2(n+1)(\omega-1)}$	$\frac{2(\alpha+n)}{2(n+2)-\beta(n+1)}$	$\frac{2(\alpha+n)}{2(n+3)-\beta(n+1)}$
F_3	$\frac{\alpha(n+1)-n}{n+2}$	$\frac{\alpha(n+1)-2n}{n+3}$	$\frac{\kappa[\alpha(n+1)-n]}{n+1+\kappa(n+2)}$	$\frac{2[\alpha(n+1)-n]}{2(n+2)+(n+1)(\omega-1)}$	$\frac{\alpha(n+1)-2n}{2(n+3)+2(n+1)(\omega-1)}$	$-\frac{2n}{2(n+3)+2(n+1)(\omega-1)}$	$\frac{\alpha(n+1)(2-\beta)-2n}{2(n+2)-\beta(n+1)}$	$\frac{\alpha(n+1)(2-\beta)-4n}{2(n+3)-\beta(n+1)}$
$F_3 - F_2$	$\frac{n(\alpha-2)}{n+2}$	$\frac{n(\alpha-3)}{n+3}$	$\frac{n[\alpha\kappa-(2\kappa+1)]}{n+1+\kappa(n+2)}$	$\frac{\alpha[2n-(n+1)(\omega-1)]-4n}{2(n+2)+(n+1)(\omega+1)}$	$\frac{\alpha[2n-(n+1)(\omega-1)]-6n}{2(n+3)+2(n+1)(\omega-1)}$	–	$\frac{\alpha[2n-\beta(n+1)]-4n}{2(n+2)-\beta(n+1)}$	$\frac{\alpha[2n-\beta(n+1)]-6n}{2(n+3)-\beta(n+1)}$
Experimental verification	[ref. 1]	[ref. 6]	[ref. 3]	[ref. 1]	[ref. 5]	[ref. 2]	[ref. 1]	–

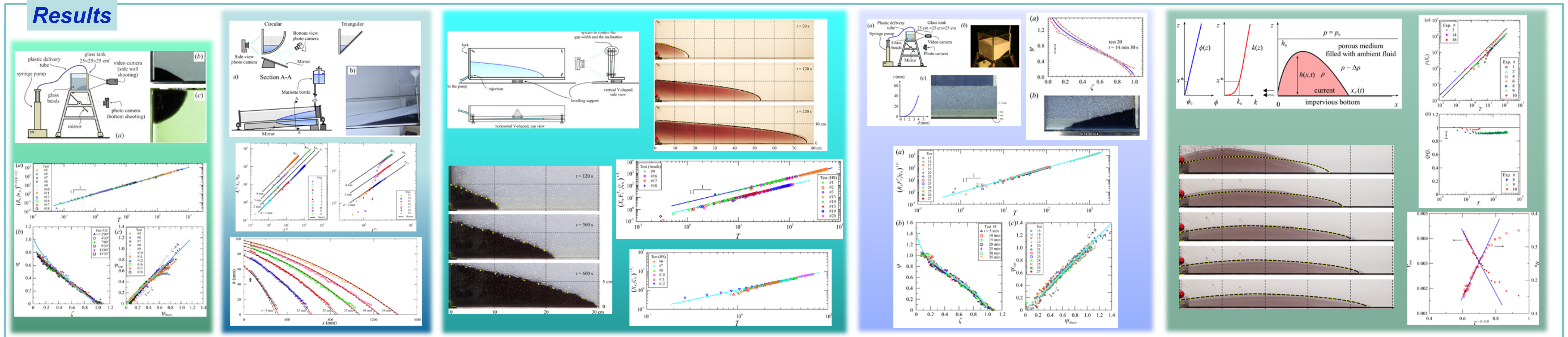
Confined flow

- Injected mass $M \propto T^{\alpha}$
- Velocity of the current $U_1 \propto T^{F_2-1}$
- Position of pressure front $L \approx T^{F_2}$
- Pressure increment $P - P_e \approx T^{F_3}$

	Constant injected mass [ref. 13]	Variable injected mass, k heterogeneity [ref. 11]
Parameters	n, d	α, n, d, β $k \propto x^{\beta}, r^{\beta}$ homogeneous if $\beta = 0$
$F_2 - 1$	$-\frac{1+d(1-n)}{1+n+d(1-n)}$	$\frac{2\alpha(n+1)-2[1+d(1-n)]+\beta(n+1)}{2[1+n+d(1-n)]-\beta(n+1)}$
F_2	$\frac{n}{1+n+d(1-n)}$	$\frac{2[\alpha(1-n)+n]}{2[1+n+d(1-n)]-\beta(n+1)}$
F_3	$-\frac{dn}{1+n+d(1-n)}$	$\frac{\alpha(2-\beta)(n+1)-2dn}{2[1+n+d(1-n)]-\beta(n+1)}$



Results

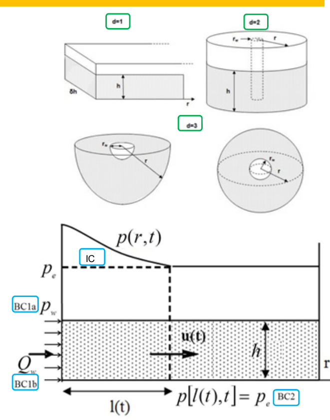


Confined flow

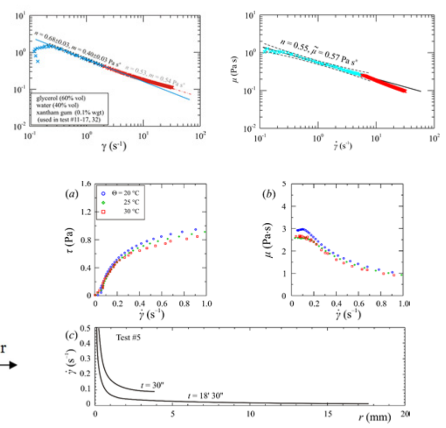
- Generalized geometry governed by d: 1 = plane, 2 = radial, 3 = spherical;
- r = generalized coordinate; t = time
- Motion driven by injection of a given fluid mass in an ambient fluid of given ambient pressure p_e
- Shear-thinning fluid (n<1)
- Advancing pressure front of position $l(t)$
- The pressure within the medium p is determined as $p(r,t)$
- Mass injected at the system boundary $m \propto t^{\alpha}$
($\alpha = 0$ constant volume, $\alpha = 1$ constant flux injection)
- Heterogeneity along direction of propagation given by $k \propto z^{\beta}$ ($\beta = 0$ homogeneous)

Self-similar solutions govern the long-time evolution of the pressure. Scalings for the front position and pressure are

$$L \approx T^{F_2} \qquad P - P_e \approx T^{F_3}$$



Rheometry



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