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Granular Streams Rheology and Mechanics

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Abstract
In the present paper we describe a model of the transition from a quasi static to dynamic regime in a granular stream. The model was developed using the results of experiments carried out on a rotating drum partially filled with sand grains or glass beads; the experiments provide information about rheology through grain velocity profiles and through the grain velocity covariance tensor. The model relies on several assumptions: we express the frictional stress component, due to prolonged contacts between particles, with a Coulomb law, assuming that the friction angle is equal to the true friction angle between the particle surfaces at contact plus the angle between the mean contact plane and the shearing plane. The difficulties involved in measuring the volume concentration of the grains with the necessary precision and the substantial impossibility of checking the results, suggest a closure based on the contact angle. We assume that the average contact angle in the frictional regime is the same as the average collisional angle in the collisional regime. The collisional contribution to the global stress is expressed as a function of the mean concentration, the local grain velocity gradient and the average contact angle between shearing layers (we implicitly assume that collisions between particles are binary and that multiple contacts between particles in movement generate friction); the kinetic contribution is not taken into account because of its minor relevance at high concentration.

The numerical model gives a satisfactory reproduction of the experimental grain velocity profiles.

1 Introduction

The present understanding of the behaviour of grain particles in granular flows is still not perfect, especially if compared with the developed studies in the field of Newtonian fluids or molecular gases; nevertheless the theories about the rheology of dry grain particles are being confirmed and seem to be qualitatively correct, despite the restrictive hypothesis.

At present none of the available models cover the full range of possible regimes, i.e. kinetic, collisional, frictional or quasi-static, and further experimental investigations are necessary to understand the physical processes, especially in the quasi-static regime.

We built a rotating drum able to generate a continuous stationary grain flow representing a variety of possible regimes of motion (Fig.1). We measured the grain velocity and the grain pseudo-temperature (the grain velocity covariance) with a LDA system, obtaining information on the flow profiles through video image analysis. A detailed description of the experiments has been widely presented in several reports of the EU funded Debris Flow Risk project as well as in several papers mentioned in the references and will be shortly summarised here.

By observing the typical grain velocity profiles of dry granular streams (Fig.2), a convex portion near the bottom, a quasi-linear profile in the mid area and a generally concave one in the upper area, between the envelopes of the fluctuations are all-evident. Several authors have obtained similar profiles (Savage, 1979; Ishida & Shirai, 1979; Takahashi, 1981; Knight, 1983). Interpreting such behaviour can be carried out within the framework of Bagnold's resistance law if the interparticle fluid is viscous; a different mechanism has to be invoked if the interparticle fluid is air.

In the present paper we develop a model of the stress transfer mechanism in a granular stream over an erodible bottom, focusing on the transition between dynamic and quasi static conditions extending the model of Ancey (1997) and disregarding the fluctuations induced by free surface perturbations.

2 Experimental set up and results

The rotating drum is a cylinder with an inner diameter of 390 mm and 131 mm wide. The cylinder is placed on a pair of friction rollers and is kept rotating about its axis at a constant velocity (Fig.1). The grain flow is on average stationary, with visible and recordable pulsation of the stream thickness and of the grain velocity, having a period of a few seconds. The amplitude of the pulsation, in

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the stable cases reported in Tab.1, varies from 20 to 10% of the stream thickness, except for sand grains 0.84-1.19 mm at 5 rpm rotation rate (see Tab.2). Therefore it is assumed that the perturbation does not influence the transition area. Moreover near the surface the volume concentration of the grains is reduced respect to its mean value in the stream; the increment in grain velocity has as counterpart a reduction in the bulk density of the grains, with a variation of the flux of momentum respect to absent perturbation case almost negligible. The granular system is far from the stick-slide motion, that disappears at rotation rate higher than 1 rpm for sand grains and is almost not present for glass beads.

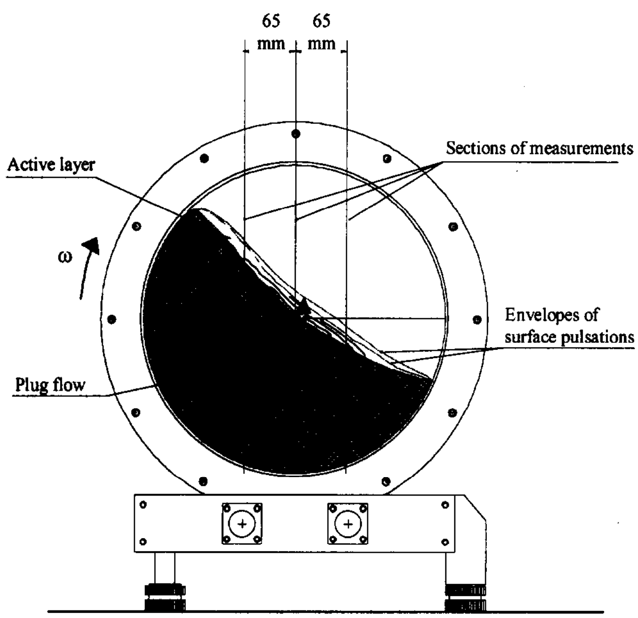

Fig. 1 Front end of the rotating drum and different regions of motion.

Table 1. Tested materials and test conditions. Sand grains B' have been obtained by sieving the material B. ϕ_r is the maximum slope angle before avalanching, ϕ_s is the slope angle after avalanching, v_0 is the maximum grain volume concentration obtained under vibration.

Material	ρ_s (Kg/m^3)	ϕ_r(°)	ϕ_s(°)	v_0	ω (rpm)
Sand grains 1.2-1.7 mm	2520	35.1± 0.2	30.9± 0.6	0.58	5
Sand grains 0.1 mm	-	-	-	-	unstable
Sand grains 0.2-0.4 mm (B)	-	-	-	-	unstable
Sand grains 0.3-0.4 mm (B')	-	-	-	-	unstable
Sand grains 0.84-1.19 mm	2530	33.5± 0.2	30.1± 0.4	0.57	2-5
Glass beads 0.2-0.3 mm	-	-	-	-	unstable
Glass beads 0.3-0.4 mm	2410	23.3± 0.2	22.5± 0.2	0.62	2-5

The drum was operated at 2 and 5 rpm (see Tab.1). Grain velocity measurements were carried out in three vertical sections using LDA in back-scatter through the glass: one section through the axis of rotation, the other two at a distance of 65 mm on both sides; the step was of 1 mm. The granular temperature was evaluated from the variance of the velocity signal. The central section represents a non-accelerated stream, while in the upper and in the lower sections the stream accelerates and decelerates respectively. The geometric characteristics of the stream are evaluated through visual observation and through the analysis of the grain velocity profiles.

Table 2. Geometric characteristics of the stream for the 'stable' cases of Table.1

Material	Computed flowing layer thickness	distance between the free surface envelopes	ω (rpm)
Sand grains 1.2-1.7 mm	22.7d	4.7 d	5
Sand grains 0.84-1.19 mm	18.2 d	11.8 d	5
Sand grains 0.84-1.19 mm	14.6 d	2.7 d	2
Glass beads 0.3-0.4 mm	34.0 d	5.7 d	5
Glass beads 0.3-0.4 mm	37.5 d	4.0 d	2

The different regions of motion are reported in Fig.2. The plug flow is well defined, with the grain velocity vector tangent to coaxial circles and the modulus proportional to the distance from the axis. Every distortion of this motion reveals the influence of the active layer or of some secondary phenomena, such as the percolation of small particles. The distortion point of the grain velocity profile is denoted as computed bottom and is taken as an indicator of the maximum extent of the active layer.

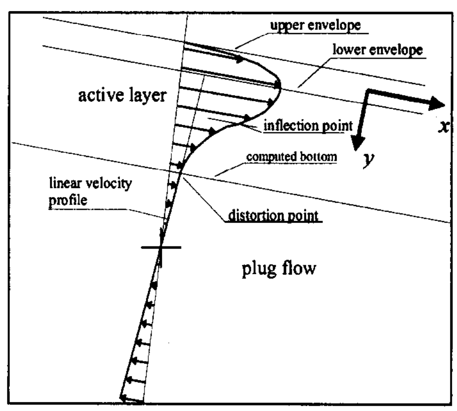

Fig. 2 Qualitative grain velocity profile in the plug flow and in the active layer and reference system. The periphery of the rotating drum is not reported.

The active layer can be divided into a lower part where the grain velocity gradient decreases with the increasing depth (and shear stress) that we call quasi static (see Fig.5), and an upper part where both grain velocity gradients and stress increase with depth, that we call the active layer. In Figs. 3 and 4 the grain velocity profile and the pseudo temperature profile for glass beads 0.3-0.4 mm at 2 rpm is shown, with error bands equal to the standard deviation calculated according to the best fitting of six velocity measurements at each point. The plotted data refer to the full data set and include the effects of the pulsation: the strong regularity of the free surface perturbation justifies an error band in the perturbation area of the same order of the error bands in the stream (were the perturbation effect is strongly reduced), whereas a reduction of the mean value of the velocity is present in the same area for perturbation travelling up-stream (it appears as an increment for perturbation travelling down-stream).

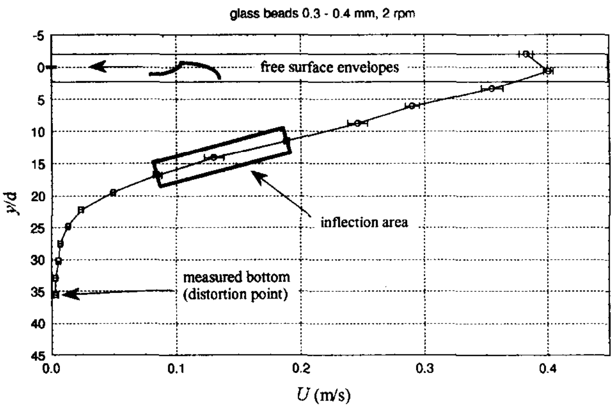

Fig. 3 Grain speed in the mid section ; 0.3-0.4 mm glass beads, 2 rpm.

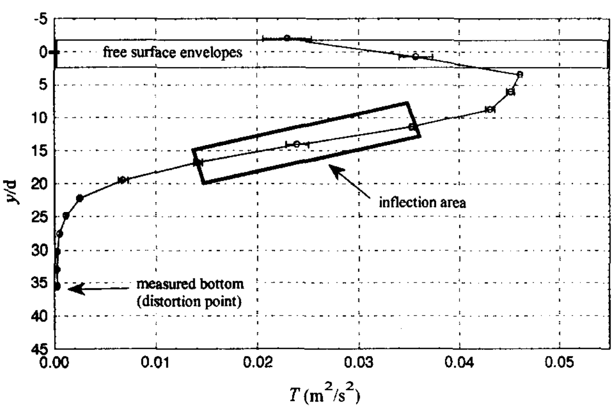

Fig. 4 Pseudo temperature in the mid section for 0.3-0.4 mm glass beads, 2 rpm.

The inflection point is difficult to detect through experiment but it corresponds to a rapid drop in the granular temperature; its position can be assumed to be where the temperature drops below 50% of the maximum value.

Similar results were achieved for all the test conditions reported in Tab.1.

3 A model of the transition from a dynamic to a quasi static regime

Our experiments clearly show that the collisional regime is limited to a few grain layers near the free surface whereas the quasi static and frictional regime are characteristic of most of the stream depth (see Fig.5).

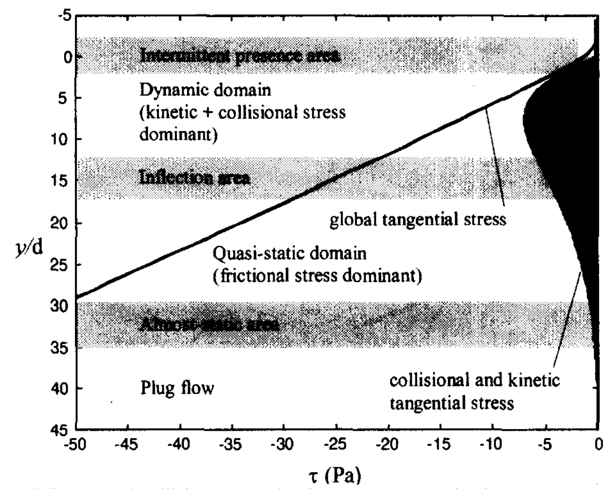

Fig. 5 Tangential collisional and kinetic stress computed using the experimental output in the model of Lun et al. (1984), mid section, 0.3-0.4 mm glass beads, 2 rpm.

This means that it is necessary to properly model the transition between the collisional and frictional regimes and the frictional regime to fully understand granular stream dynamics. In granular flows an important state variable is the volume concentration of grains: a small variation in volume concentration can radically change the stress transfer regime. Unfortunately it is really difficult to measure concentration with the necessary precision and even though we recognise the essential role it plays, we have preferred to model a parameter, the average contact angle β between layers of grains (see Fig.6). In reality, this parameter is a consequence of the shear rate regime and of the concentration and in any case is not a cause but an effect.

The average contact angle is a measurement of contact geometry. The organisation of the contacts is the responsible (or an indicator) for the stress transfer mechanism: a permanent contact network between particles transfers stress in static or quasi-static regime whereas instantaneous contacts transfer stress in dynamic regime (collisional regime). The average contact angle assumes undefined values for a perfect isotropy of contacts, as in the two extreme situations of grains at rest without distortion stress or of grains in random motion in a dense gas state. A non-isotropic stress applied to grains initially at rest polarises the contacts and induces finite values of β.

In presence of pure shear the average contact angle tends towards 45° and decreases with increasing pseudotemperature, i.e. particle agitation due to collisions reduces the anisotropy of contacts.

In a quasi static regime, with a finite deformation of the medium and with a mean contact plane which does not coincide with the sliding plane, we assume that most of the stress is transferred according to Mohr-Coulomb law with a friction angle that is equal to the particle surface friction angle and the mean contact angle.

$$\tau_f \le \sigma_f \tan(\varphi_o + \beta) \tag{1}$$

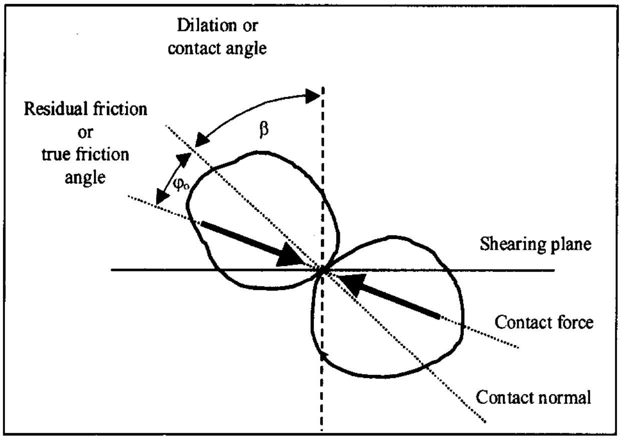

Fig. 6 Description of the geometry at contact.

The true friction angle φ_o depends on the roughness of the surface, the grain material or the presence of a lubricant fluid; there is a strong reduction when shearing starts, due to the collapse of grain-grain interlocking. The average contact angle tends to increase with the stress level and to decrease with the shear rate. The variability of the apparent friction angle with the stress level and with the particle array is well known (Leps, 1970). In many triaxial tests the friction angle decreases with an increasing void index and increasing stress; this dependence is justified by particle deformation under stress whereas in the analysed situation the stress level is quite small and particle deformation is negligible.

Our reference scheme for interpreting the granular system behaviour in quasi-static conditions is reported in Fig.7. The critical state can be reached through the dilatation of densely packed soils or by the contraction of loosely packed soils. We analyse the behaviour of the system when the friction angle increases from its minimum dynamic value to its static value (the dotted path in Fig.7); the variation of the apparent friction angle is of a few degrees but the gradient in grain volume concentration is so small that it can hardly be modelled with confidence.

We still need to assume that stress is partially transferred by collisions otherwise the velocity of the medium would be undefined: the collisional components of stress are modelled according to Ancey (1997), who implicitly assumes a local balance of pseudothermal energy (associated with particle fluctuations), i.e. the energy from the large scale motion is locally dissipated by collisions.

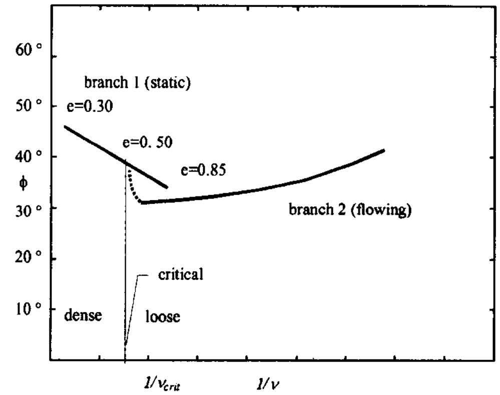

Fig. 7 Internal friction angle vs. grain volume concentration for sand grains.

The momentum balance equations are:

$$\begin{aligned} \sigma_c + \sigma_f &= \int_0^y \rho_s v g \cos\theta \, dy + c_1 \\ \tau_c + \tau_f &= \int_0^y \rho_s v g \sin\theta \, dy + c_2 \end{aligned} \tag{2}$$

σ_c and σ_f are the normal collisional and frictional stress, τ_c and τ_f are the tangential collisional and frictional stress, v is the particle concentration, g is gravity acceleration, θ is stream inclination.

The expressions for the collisional components are (Ancey, 1997):

$$\begin{aligned} \sigma_c &= v\rho_s d^2 f_1(\beta)\left(\frac{\partial U}{\partial y}\right)^2 \\ \tau_c &= v\rho_s d^2 f_2(\beta)\frac{\partial U}{\partial y}\left|\frac{\partial U}{\partial y}\right| \end{aligned} \tag{3}$$

where d is the particle diameter, f_1 and f_2 are functions of the elastic restitution coefficient e and of the average contact angle β. We assume that the average contact angle in a frictional stress regime (multiple contacts between particles) is equal to the average collision angle in a collisional stress regime (binary frictionless collisions between particles), i.e. the statistics of contacts between particles is not influenced by the nature of the two particles' contacts with the other surrounding particles, at least if the system is close to the critical condition.

The system of equations is:

$$\begin{aligned} &\bar{v}\rho_s d^2 f_1(\beta)\left(\frac{\partial U}{\partial y}\right)^2 + \sigma_f = \bar{v}\rho_s g y \cos\theta \\ &\bar{v}\rho_s d^2 f_2(\beta)\frac{\partial U}{\partial y}\left|\frac{\partial U}{\partial y}\right| + \\ &\sigma_f \tan(\varphi_o + \beta) = \bar{v}\rho_s g y \sin\theta \end{aligned} \tag{4}$$

where $\bar{\nu}$ is the mean grain volume concentration. Two suitable expressions for the functions f_1 and f_2 are (Ancey, 1977):

$$f_1(\beta)=\frac{(1+e)}{4}\beta,\ f_2(\beta)=\frac{(1+e)}{4}\beta^2. \qquad (5)$$

We need a closure for β. We assume that β represents the average collision angle for binary collisions (≈ collisional regime) and the average contact angle for particles with multiple contacts (≈quasi static regime); in the latter case β cannot be greater than the dilation angle. Using the concepts expressed at the beginning of this chapter, we can assume that $\beta=f(\rho_s, d, I_1, J_2, E)$, where ρ_s is the mass density of the grains, d is the grain diameter, I_1 is the first invariant of stress tensor and $J_2 \equiv \sqrt{2\mathbf{D}:\mathbf{D}}$ is the second invariant of the velocity of deformation tensor, E is the Young module of the grain material. The dependency on the Young's module reflects the behaviour of grains at rest in a quasi-static regime, where the stress level is strictly related to the stress response of a single grain. Introducing the two non-dimensional parameters $(\rho_s d^2 J_2^2/E)$ and (I_1/E), β can be expressed as $f(\rho_s d^2 J_2^2/E, I_1/E)$; the first parameter is the ratio: (scale of the collisional component of the stress) / (scale of the static stress). The second is the ratio of the scale of the global stress to the scale of the static stress. Note that gravity is not directly included but still acts as one of the sources of the stress.

Assuming $\beta=(\rho_s d^2 J_2^2/E)^a (I_1/E)^b$ for a=-1/2 and b=2 we can obtain the same structure as the expression proposed by Ancey (1997).

The assumed closure relation is:

$$\beta=\min\left(\text{dilation angle},\ s\frac{I_1^2}{(\rho_s d^2)^{1/2} E^{3/2} J_2}\right) \qquad (6)$$

where s is a positive constant. According to this expression in simple shear flow β increases with the stress level and decreases with the shear rate, i.e. with the square root of the pseudo temperature in local balance conditions. Note that several structures of the function f are allowed and the suggested one is proposed in agreement with Ancey, (1997); the correct one should be found through experimentation. Some dependence on granular shape should also be included, but at present the only available information is reflected in the friction angle (higher for irregular shaped sand grains than for glass spheres) and in the difference between ϕ_r and ϕ_s (see Tab.1).

In a 2-D granular stream the first invariant of the stress is assumed to be proportional to the normal stress along y. It means that we can assume a constant ratio between the normal stress along the two other directions and the normal stress along y, independent of the regime, with all the coefficients included in s:

$$\beta=s\left(\rho_s \nu g y^2 \cos^2\theta \Big/ \frac{\partial U}{\partial y}\right) \qquad (7)$$

By substituting the frictional normal stress from the first into the second equation of the system (4), using the closure reported in Eq.(5), (6) and (7), we can obtain a non linear differential equation in the variable U (the volume concentration ν of the grains is assumed to be constant).

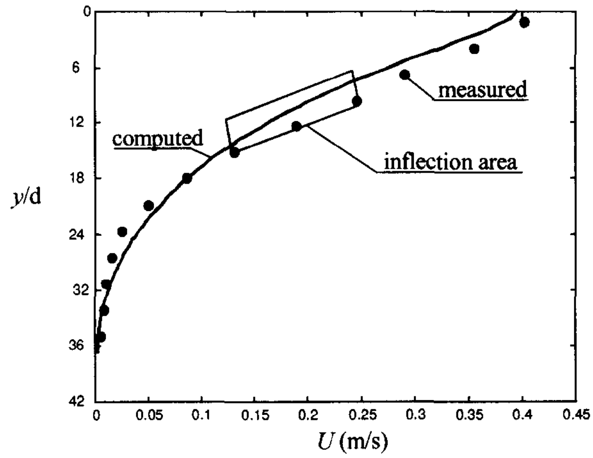

Fig. 8 Grain velocity profile in the mid section, calculated and measured. Glass beads 0.3-0.4 mm, 2 rpm.

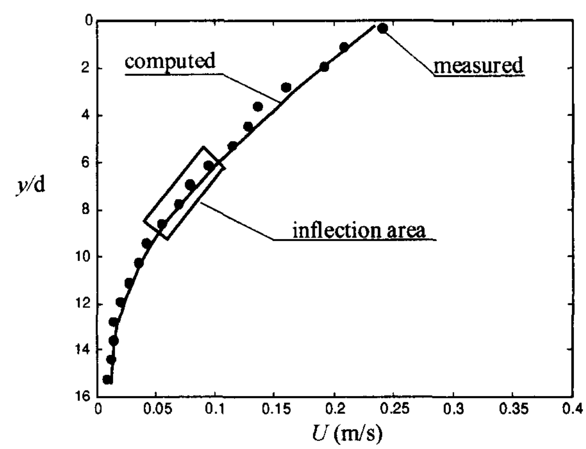

Fig. 9 Grain velocity profile in the mid section, calculated and measured. Sand grains 0.84-1.19 mm, 2 rpm.

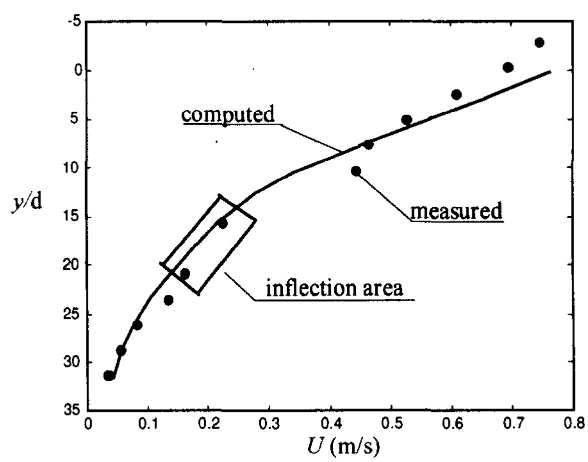

Fig. 10 Grain velocity profile in the mid section, calculated and measured. Glass beads 0.3-0.4 mm, 5 rpm.

For the first attempt we imposed non-slip conditions and a zero shear rate at the bottom. The integration is limited to the domain between the bottom and a couple of grain diameters below the free surface; above this the continuum model is no longer valid. Figs.8, 9 and 10 show the calculated and measured grain velocity profile for glass beads and sand grains at different rotation rates. The restitution coefficient e was not measured; it was assumed equal to 0.88 for glass beads and to 0.6 for sand grains.

The maximum difference between computed and measured grain velocity is less than 10% in the region from the bottom to the inflection area, except for glass beads at 5 rpm (see Fig.10). It should be noticed that in this last case, as well as for sand grains 0.84-1.19 mm at 5 rpm and sand grains 1.2-1.7 mm at 5 rpm, only two LDA measurements per point were carried out instead of six measurements.

Results almost similar to those plotted in Figs. 8, 9 and 10 were obtained for sand grains 1.2-1.7 mm at 5 rpm. Sand grains 0.84-1.19 mm at 5 rpm were not modelled because of the relevant amplitude of the perturbation respect to the stream thickness (see Tab.2).

3 Discussions

By observing the velocity profiles of a dry granular stream three essential regions can be detected: near the bottom the shear decreases with increasing shear stress, in the mid region the shear rate is almost constant, near the free surface there is an inversion or an enhancement of the grain velocity, limited to a few grain diameters, due to surface perturbations. The regions correspond to a quasi-static stress transfer and a collisional stress transfer (the dynamic area).

The inflection point, separating the quasi-static and the dynamic area, is approximately at the level where the granular pseudo temperature drops 50% of its maximum value.

In modelling the transition from a collisional regime to a quasi-static regime we did not use grain volume concentration as a state variable; instead we modelled the average contact angle. That is equivalent to assuming the organisation of the contacts among the particles as the indicator of the stress transfer regime: in a static regime a permanent contact network transfers the forces; in a dynamic regime collisions with a short contact time transfer the stress. Kinetic stress transfer, due to a flux of momentum similar to Reynold's flux of momentum in turbulence modelling, is limited to a thin layer near the surface and is thus negligible.

Ancey (1997) has already used the average contact angle for modelling the collisional regime; in our approach it is also used to describe the frictional regime in a Mohr-Coulomb framework. The closure is then transferred to the description of the average contact angle. We describe it as a function of the stress level and of the local velocity of deformation.

In general turbulence development is expected in the granular stream, but in the tested situations the stream thickness is of a few grain diameter: macro turbulence has no space to develop. The present model is focused on the description of a regime were friction dumps turbulence, quite similar to the viscous sub-layer in a fluid in turbulent regime. As a consequence turbulence can be neglected as a first approximation.

4 Conclusions

A numerical model based on the assumption of a Mohr-Coulomb law and a collisional law coexistent in the whole active layer, both expressed in terms of the average contact angle, can give a satisfactory reproduction of the experimental grain velocity profile, with a computed maximum error in grain velocity of 10% in the region from the bottom to the inflection area.

Acknowledgements

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