PERTURBATION PROPAGATION IN COUNTERCURRENTS OF NON-NEWTONIAN FLUIDS

Sandro Longo [1], Luca Chiapponi [1], Fabio Addona[1], Nicolò Merli[1], Vittorio Di Federico[2]

(1) Dipartimento di Ingegneria Civile e Architettura, Università di Parma.; (2) Department of Civil, Chemical, Environmental, and Materials Engineering, Università di Bologna

KEY POINTS

- *Our theoretical-experimental analysis of buoyancy flows in a volcanic chimney, conceptualized as a vertical circular duct, identifies counterflow regimes of two non-Newtonian fluids of different density.*
- *We carry out an analysis of the speed of propagation of perturbations, identified as sudden changes in density and in the rheological parameters.*
- *Results indicate the importance of the variability of rheological parameters in generating perturbation waves also towards the magma reservoir, which can trigger subdued eruptive processes.*

1 INTRODUCTION

The magma dynamics in volcanic chimneys having a near-cylindrical shape, has been interpreted in the light of flow and thermodynamic processes which, even in the absence of eruptive activity, generate a continuous degassing process favoured by the counter current recirculation of a lighter fluid along the cylinder axis and a denser fluid in its peripheral ring. At or near the crater, the lighter fluid cools after releasing its gases and precipitates downwards, moving along the walls. The source of energy is the magma reservoir, which heats the nearby fluid and causes its density to decrease. Several other processes then take place, such as internal degassing due to pressure reduction, which causes both a decrease in average density and an increase in yield strength due to crystal formation. The latter phenomenon also leads to the joint variation of the consistency and fluid behaviour indexes, since the presence of gas bubbles in a high shear regime results in a shear thinning behaviour, even if the matrix fluid is Newtonian.

A first aspect of interest is the flow rate generated by buoyancy, which is the result of i) the rheology of two (generally different) fluids, ii) the difference in density and the consequent buoyancy of the flow, and iii) the geometry of the currents as described by the radius of the circular section occupied by the less dense fluid in relation to the total radius of the circular duct. This aspect has already been analysed for the case of an external fluid described by the Herschel-Bulkley (HB) model and an internal, less dense, Newtonian-type fluid in *Longo et al.* (2022). A second aspect relates to the direction of propagation of the perturbations, the latter schematised, e.g., as a reduction in the density of the inner fluid, for example when degassing begins. In reality, each perturbation, represented by a change in one of several parameters involved in the dynamic description of the flow, requires an analysis of the direction of propagation. Particularly relevant are those perturbations propagating towards the magma reservoir which, as they accumulate, could trigger a paroxysmal eruptive process with catastrophic amplification of degassing and magma ascent in the vent. Disturbances propagating towards the crater mouth, on the other hand, are destined to attenuate near the magma-atmosphere interface.

A first step in this direction was taken by *Peng et al.* (2022) who, in the configuration described above but with Newtonian fluids, modelled the propagation of the internal fluid radius, density and viscosity and concluded that the former can take on both positive (upward) and negative (downward) values depending on the viscosity ratio and the internal fluid radius. This means that, under certain flow conditions, a fluctuation in the cross-sectional area occupied by the fluids can propagate towards the magma reservoir.

In this paper, we analyse the process under the assumption that both fluids are HB, considering all possible combinations. We then proceed to study the speed of propagation of a disturbance represented by a sudden stepwise increase of the variable of interest from the centre of the vertical cylindrical duct.

2 THEORY

We consider the schematic in Figure 1a, where a cylindrical coordinate system is adopted.

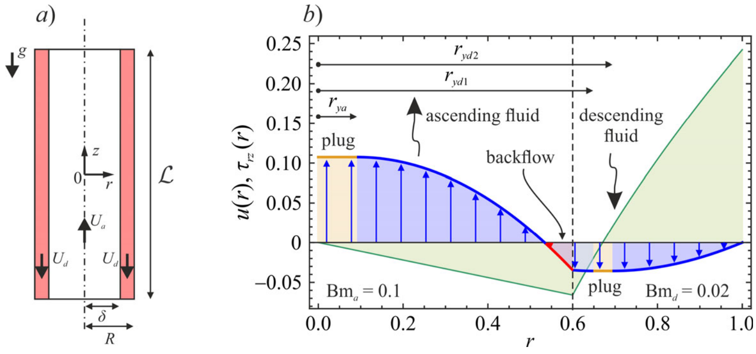

Figure 1. a) Schematic, and b) velocity and stress profiles.

For both the ascending (*a*) and descending (*d*) fluid we assume a HB model, reading

$$|\boldsymbol{\tau}| \leq \tau_{y(a,d)} \quad \text{if} \quad |\dot{\boldsymbol{\gamma}}|_{a,d} = 0, \tag{1a}$$

$$\boldsymbol{\tau} = \left(\boldsymbol{\mu}_{(a,d)} |\dot{\boldsymbol{\gamma}}|_{a,d}^{n(a,d)-1} + \frac{\tau_{y(a,d)}}{|\dot{\boldsymbol{\gamma}}|_{a,d}}\right) \dot{\boldsymbol{\gamma}}_{\mathrm{a,d}} \quad \text{if} \quad |\dot{\boldsymbol{\gamma}}|_{\mathrm{a,d}} \neq 0, \tag{1b}$$

where $\mu_{(a,d)}$ is the consistency index, $n_{(a,d)}$ is the fluid behaviour index, $\tau_{y(a,d)}$ is the yield strength, $|\dot{\boldsymbol{\gamma}}|_{a,d} = \sqrt{(\dot{\boldsymbol{\gamma}}_{a,d} : \dot{\boldsymbol{\gamma}}_{a,d})/2}$, and $|\boldsymbol{\tau}| = \sqrt{(\boldsymbol{\tau} : \boldsymbol{\tau})/2}$ are measures of the shear rate and shear stress, respectively. As special cases, $\tau_y = 0$ describes an Ostwald-deWaele fluid, $\tau_y \neq 0, n = 1$ describes a Bingham fluid, and $\tau_y = 0, n = 1$ describes a Newtonian fluid. By assuming incompressibility and $R \ll L$, under the hypothesis of lubrication flow the velocity profile can be computed with the boundary conditions of no-slip at the walls and null shear stress at the axis. The dimensionless form of the equations includes a consistency index ratio, a density contrast and dimensionless yield strength in addition to the fluid behaviour index for the ascending and descending fluids. Note that in the most general case, a central plug develops at the axis for the ascending fluid, and a ring plug develops for the descending fluid, depending on the yield strength values. The ascending and descending flow rates depend on the value of δ, the radius of the internal fluid cross section, which is not unique and can be inferred based on energy consideration and through experiments (see *Longo et al., 2022*).

The next step is the computation of the celerity of different kinds of perturbations, e.g., a stepwise variation of density described by a Heaviside function. The continuity equation for the internal ascending current can be expressed as

$$\frac{\partial \pi \delta^2}{\partial t} + \frac{\partial Q}{\partial z} = 0, \tag{2}$$

where Q is the flowrate of the ascending fluid (equal to the flowrate of the descending fluid). Eq. (2) is a kinematic wave which con be explicitly written as

$$\frac{\partial \delta}{\partial t} + \left(\frac{1}{2\pi\delta} \frac{\partial Q}{\partial \delta}\right) \frac{\partial \delta}{\partial z} = 0, \tag{3}$$

where the term in round braces is the celerity of the perturbation. Since the flux is a function of several variables, $Q = Q(\delta, n_a, n_d, Bm_a, Bm_d, \phi, \Psi)$, where Bm_a, Bm_d are the dimensionless yield strengths, ϕ is the consistency index ratio and Ψ is the density parameter, we can estimate the variation of Q due to the perturbation of each variable as (limiting our analysis to the internal ascending fluid variation)

$$\mathrm{d}Q = \left(\frac{\partial Q}{\partial \delta}\right)_{n_a, Bm_a, \phi, \Psi} \mathrm{d}\delta +$$

$$\left(\frac{\partial Q}{\partial \mathrm{n_a}}\right)_{\delta, Bm_a, \phi, \Psi} \mathrm{d}n_a + \left(\frac{\partial Q}{\partial \mathrm{Bm_a}}\right)_{\delta, n_a, \phi, \Psi} \mathrm{d}Bm_a + \left(\frac{\partial Q}{\partial \phi}\right)_{\delta, n_a, Bm_a, \Psi} \mathrm{d}\phi + \left(\frac{\partial Q}{\partial \Psi}\right)_{\delta, n_a, Bm_a, \phi} \mathrm{d}\Psi, \tag{4}$$

or

$$\mathrm{d}Q = c_\delta \mathrm{d}\delta + c_{n_a}\mathrm{d}n_\mathrm{a} + c_{Bm_a}\mathrm{d}Bm_\mathrm{a} + c_\phi \mathrm{d}\phi + c_\Psi \mathrm{d}\Psi, \tag{5}$$

where the flux variation is associated to the contribution of the celerity of each changing variable.

Once the celerity of perturbation is known as a function of δ, a finite difference scheme is adopted to estimate the corresponding perturbation evolution.

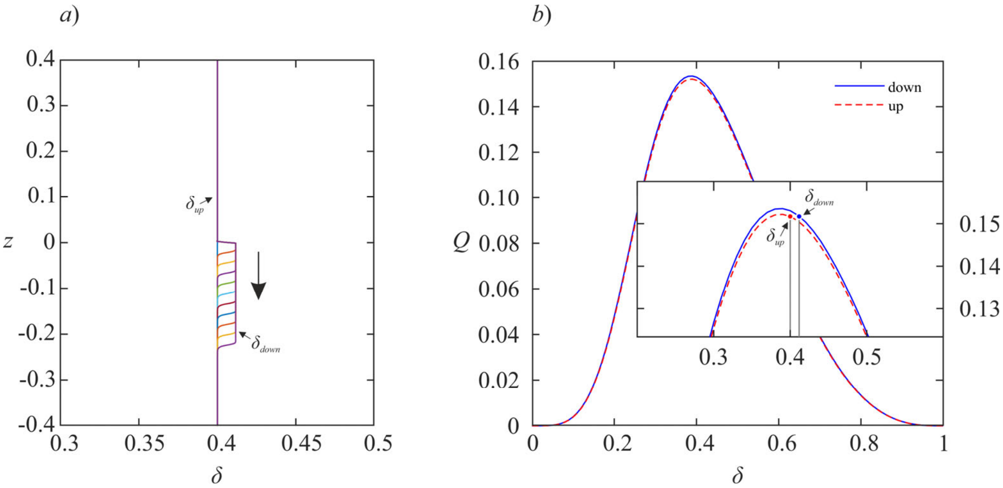

Figure 2. a) Time evolution of the radius of the ascending fluid at $T = 0, 0.5, \ldots, 5$, and b) fluxes in the upper domain $z > 0$ and in the lower domain $z < 0$. Computations with $n_a = n_d = 1$, $\phi = 100$, $\Psi = 1$, $Bm_a = Bm_d = 0$ for $z < 0$, and with $n_a = n_d = 1$, $\phi = 100$, $\Psi = 1$, $Bm_a = 0.1$, $Bm_d = 0$ for $z > 0$.

Figure 2a shows the propagation of the perturbation downward, corresponding to an increment of Bm_a from 0 to 0.1 in the upper domain $z > 0$, resulting in an increment of δ from $\delta_{up} = 0.4$ to $\delta_{down} = 0.42$. Each plot refers to $\Delta T = 0.5$. Figure 2b shows the fluxes corresponding to the yield strength variation and the core thicknesses with the same flux.

Some preliminary experiments have been carried out with a fast reduction of the ascending fluid density obtained by adding citric acid to the fluid and then injecting a similar mixture added with sodium bicarbonate with a needle at $z = 0$. Consequently, the production of carbon dioxide induces a fast reduction of the (average) density of the ascending fluid corresponding to $\Psi > 1$ in $z > 0$. The evolution is shown in Figure 3ab.

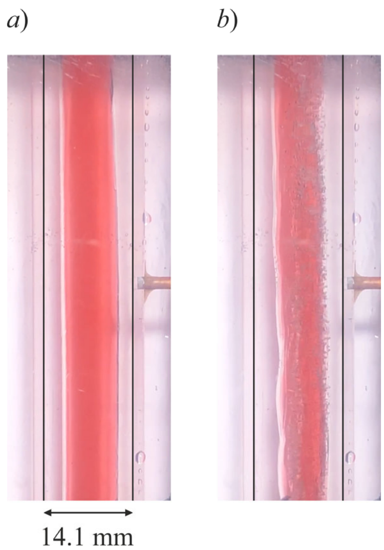

Figure 3. Evolution of the radius of the internal fluid due to a sudden reduction of density. Vertical pipe with internal radius R = 7.05 mm. a) Initial configuration, with $\rho_a = 1225.3$ kg /m^3, $\rho_d = 1257.7$ kg /m^3, μ_a = 10-3 Pa s, $\mu_d = 0.329$ Pa, $Bm_a = 0$, $Bm_d = 0$ and $\delta_0 = 0.65$. b) Configuration after reducing the ascending fluid density, with $\delta_1 = 0.56$. The ascending fluid is glycerol plus water, the internal fluid is water added with phosphates, sodium chloride and citric acid. The injected fluid is water added with phosphates, sodium chloride and sodium bicarbonate having the same density of the internal fluid.

REFERENCES

Longo, S., Chiapponi, L., Petrolo, D., Bosa, S., Di Federico, V., 2022. Ascending non-Newtonian long drops in vertical tubes. Journal of Fluid Mechanics, Vol. 950, A1, 1—45.

Peng S, Picchi D, Suckale J. Disrupt the upper or the lower conduit? The dual role of gas exsolution in the conduits of persistently active volcanoes. Journal of Fluid Mechanics. 2022;942:A24.