# Physically based model of soil erosion and pollutant dynamics at a basin scale

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## Abstract

In this paper we describe a conceptual model for evaluating sediment transport and pollutants dynamics (BOD, P and N) at a basin scale. The sediment transport model is physically based; the pollutant dynamics model is based on an integral and simplified form of the complex equations. The model was numerically implemented and applied to four catchments in the Po River Area, with extensions ranging from 460–2150 km$^2$. The topology is described by DEM with cells of 250x250 m$^2$. The time step is 24 h. The model was calibrated using data measured in the main streams in the period 1993–1999.
*Keywords: pollutant dynamics; sediment transport; basin management.*

## 1 Introduction

The study "Analysis of floods in relation to factors influencing nourishing compounds in the basins of the tributaries of Po River", developed by Agenzia Regionale Prevenzione e Ambiente dell'Emilia-Romagna (ARPA [1]) reports on sediment transport and quality of surface water in the main basins in Emilia Romagna, through numerical models.

The analysed chemical parameters are Nitrogen, as ammonia, nitric and nitrous, Phosphorus and BOD, hereafter defined as nutrients, and sediment transport. Due to the large time span (1993-99), and large modelled areas (more than 6000 km$^2$) of the basins (Nure, Taro, Secchia and Panaro rivers), the available information allowed only some simplified representations of the phenomena.

In the full model stream and catchment processes were separated (but linked): in the present paper only sediment transport and chemical dynamics (nutrients) in the catchment are described. A hydrologic module (ARPA [1]) is used to model fluid discharge on a daily rate.

Chemical dynamics if focussed on pollutants load due to diffused sources, starting from the estimates of the agricultural field spreadings, weather evolution, soil and crop nature. Among the inputs also sediments transport is present, because sediments carry part of the chemicals. The outputs are the daily loads transferred to the streams.

## 2 Quality model formulation

The quality model has the same space grid (250x250 m) and time step (24 h) used for the erosion model. The link between the two models is represented by eroded (or deposited) sediments which transfer the pollutants by adhesion. The present model has an intermediate complexity between the RTDR formulation by Ferraresi [3], and the models HSPF and CREAMS by Kniesel & Nicks [6]. The former represent the conceptual reference, the latter represent the pragmatic reference, but with the exclusion of numerous physics and chemical parameters, which cannot practically be evaluated in many real situations.

The detachment, the incipient motion and the transport of the chemicals, as well as the mineralization, volatility, absorption and interaction with groundwater, are described using a simplified and integral formulation, using a limited number of parameters. Each flux contributes to the mass balance of the chemicals in the cells. This simple formulation is also justified by the real complexity of morphology and of forcing terms in the equations.

The main processes included in the model are: (1) decay due to several factors; (2) suspension in function of soil erosion; (3) transport in dispersed phase in function of running water; (4) infiltration with possible subsurface flow of the chemical, in particular for Nitrogen.

(1) The decay includes all the processes which induce a progressive reduction of the chemical in the cell. We assume that the decay rate $\Delta dec_t$ is proportional to the storage $Acc_t$ through a coefficient of decay $K_{dec}$:

$$\Delta dec_t = Acc_t \cdot K_{dec} \tag{1}$$

The coefficient of decay essentially depends on soil temperature and on the vegetative stage of the crops. These two effects show a yearly variability; as a consequence, we assume $K_{dec}$ varying on a monthly base, with minor space variability effects. In practice we assume a catchment scale as dominant, with equal uniform values of the decay coefficients for all the cells inside a defined catchment.

(2) The suspension in function of soil erosion is strictly related to the process of soil erosion as described in Sec.3. The mass quantity transported as consequence of soil erosion is expressed as:

$$\Delta ero_t = E_t \cdot (Acc_t \ / \ Acc_{\max})^{a} \cdot K_{ero} \tag{2}$$

with $E_t$ = the thickness of soil daily eroded, $Acc_{max}$ is the scale of stored mass, $a$ is a parameter and $K_{ero}$ is a coefficient determined using the following expressions:

$$\begin{aligned} K_{ero} &= 0 && \text{if } E \le E_{\min} \\ K_{ero} &= a_0 + a_1 \cdot (1 - r_m) \cdot [(E_t - E_{\min})/(E_{\max} - E_{\min})]^a && \text{if } E_{\min} < E_t \le E_{\max} \\ K_{ero} &= a_0 + a_1 \cdot (1 - r_m) && \text{if } E_t > E_{\max} \end{aligned} \tag{3}$$

$a_0$ and $a_1$ are parameters and $r_m$ is the mobility ratio, related to solubility in water of the chemical under analysis. $E_{min}$ and *Emax* represent the scale of efficiency of chemical transport due to erosion.

(3) The transport in dispersed phase in function of running water is expressed using equations similar to eqns (2) and (3): the daily quantity of chemical $\Delta sol_{t\ is}$ expressed by the following equation:

$$\Delta sol_t = R_t \cdot (Acc_t / Acc_{\max})^{\ b} \cdot K_{sol} \tag{4}$$

$R_t$ is the daily mean value of the water running water depth in the cell, $b$ is a numerical parameter and $K_{sol\ is}$ a coefficient determined using the following relations:

$$\begin{aligned} K_{sol} &= 0 && \text{if } R_t \le R_{\min} \\ K_{sol} &= a_0 + a_1 \cdot r_m \cdot [(R_t - R_{\min})/(R_{\max} - R_{\min})]^b && \text{if } R_{\min} < R_t \le R_{\max} \\ K_{sol} &= a_0 + a_1 \cdot r_m && \text{if } R_t > R_{\max} \end{aligned} \tag{5}$$

(4) The infiltration $\Delta inf_t$ depends on the existence of water supply of the water-bearing stratum $Q_{inf,t}$, according to the following relation:

$$\Delta inf_t = Q_{inf,t} \cdot (Acc_t / Acc_{\max})^c \cdot (Q_{inf,t} / Q_{\max})^d \cdot K_{inf} \tag{6}$$

with $K_{inf}$ = infiltration coefficient and the exponents $c$ and $d$ are non-dimensional parameters.

The processes are formally similar for the three chemicals considered: $BOD_5$, $N_{tot}$ and $P_{tot}$. Only for the Nitrate the balance equation includes a possible contribution $\Delta sot_t$ of the hypodermic flow $Q_{ipo}$ and of the base groundwater flow $Q_{fal}$, according to the following equation:

$$\begin{aligned} \Delta sot_t = Q_{ipo,t} \cdot \left[ c_o + c_1 \left( Q_{ipo,t} / Q_{ipo,\max} \right)^e \right] + \\ Q_{fal,t} \cdot \left[ c_2 + c_3 \left( Q_{fal,t} / Q_{fal,\max} \right)^f \right] \end{aligned} \tag{7}$$

with the usual meaning of the symbols. In addition, in the model it is possible (a) to inhibit the daily field spreading of the artificial supply of chemicals if the thickness of the snow layer exceeds a specific value, with subsequent supply as soon as the thickness of the snow layer is reduced (but with a degradation process included); (b) the option to transfer chemicals from one cell to the subsequent cell using a mass response function (Rinaldo *et al.* [12]), with important effects at basin scale; (c) the option to introduce a damping function for the chemicals based on the distance between the point of detachment and the point of inlet in the stream. This option allows the simulation of stream processes instead of surface runoff ones over cells generating a permanent morphological structure upstream of brooks and rivers.

## 3 Erosion model at a basin scale

The sediment transport model adopted is physically based and relates sediment discharge to rain and subsequent surface discharge. All the used parameters have a physical meaning and can be measured in the field. It is also possible to model their variation as a consequence of management works in the basin.

The forcing in sediment transport is rain intensity and duration; the modulating factors are local land surface inclination, the intrinsic erodibility of the soil, the real management of the soil and the effects of eventual works of maintenance.

The result is erosion or sedimentation of sediments at each time step. The balance equation used is the following (Nearing *et al.* [9], [10], [11]):

$$\frac{\partial C_s y}{\partial t} + \frac{\partial C_s q}{\partial x} = k_I I^2 + k_R \left\{ B' q^n \sqrt{\sin\theta} - C_s q \right\} \tag{8}$$

$t$ = time, $x$ = space, $C_s$ = sediment concentration, $y$ = water layer thickness, $q$ = overland flow discharge per unit width, $I$ = rainfall intensity, $\theta$ = soil slope, $n$ = exponent, $K_I$, $K_R$, $B'$ = coefficients. Averaging in time, the first term on the lhs and on the rhs are negligible, and the balance equation reduces to:

$$\frac{\partial q_s}{\partial x} + k_R q_s = k_R B' q^n \sqrt{\sin\theta} \tag{9}$$

$q_s = C_s q$ = sediment discharge per unit width. The eqn (2) can be integrated in a spatial domain wherein all terms are constant, with the following result:

$$q_s = c_1 \exp\left(-k_R x\right) + B' q^n \sqrt{\sin\theta} \tag{10}$$

The constant of integration $c_1$ is obtained imposing a known value of the sediment discharge (per unit width) in input of the cell under analysis and equal to the contribution of all the contributing cells. The output sediment discharge (per unit width) is equal to:

$$q_{s\ out} = \left(q_{s\ \mathrm{in}} - B' q^n \sqrt{\sin\theta}\right) \exp\left(-k_R x\right) + B' q^n \sqrt{\sin\theta} \tag{11}$$

In order to highlight the physical meaning of the terms, eqn (2) can be written as (Foster and Meyer [4]):

$$\frac{\partial q_s}{\partial x} + \frac{q_s}{h} = D \tag{12}$$

in which *h* is the mean travel distance of detached particles and *D* is a detachment capacity. If *x* >> *h* results in $q_s = Dh$ . A detailed analysis of the saltation and movement of sediments is due to Kirby [5], who found out an expression of *h* as:

$$h = \left[\frac{\varepsilon y \rho_s}{f(\rho_s - \rho)}\right] \frac{s}{\tan\phi} \tag{13}$$

$\varepsilon$ = efficiency in trapping and transporting sediments by the water stream, *f* = friction factor, *s* = soil slope, $\phi$ = sediment friction angle. Considering that $\varepsilon$ decreases for increasing water layer thickness, the contribution in square brackets can be assumed as a constant, and results in:

$$h \propto \frac{s}{\tan\phi} \tag{14}$$

On the base of experimental results (Meyer and Monke, [8]; Kramer and Meyer, [7]) the sediment discharge can be expressed as $q_s \propto q^n s^{n+1}$; *q* = water discharge per unit width, *n* = exponent in the range 1.7-3.5. Detachment can be expressed as $D \propto (qs)^n \tan\phi$.

The dependence of the parameters on soil erodibility *K* and cropping *C* is expressed by the following equations:

$$h = c_1 \frac{s}{\tan\phi};\ D = c_2 KC (qs)^n \tan\phi \tag{15}$$

with

$$\phi = \phi_o (1 - K)(1 - C) \tag{16}$$

In order to simulate an erodibility reduction due to prolonged dry weather, the following relation is used:

$$K = \frac{1}{2} K_o + \frac{1}{2} K_o \sqrt{1 - \tanh\left(\frac{t_s}{T}\right)} \tag{17}$$

$t_s$ = number of sequential days of dry weather and *T* time scale, assumed equal to 20 days in the following. Once the output sediment discharge is known, the mass balance in the cell allows the computation of sediment erosion.

## 4 Application of the model

The quality model (applied to the four basins under test) requires the following information: (1) description of the topology of the basin in terms of cells; (2) daily rate in each cell of $BOD_5$, $N_{tot}$ and $P_{tot}$, due to human activity and to natural sources; (3) quantification, in each cell, of the hydrological and sediment transport variables involved in chemical dynamics at a basin scale; (4) description, for each cell at each time step, of the parameters.

Each basin is represented with uniform cells 250 x 250 m in size, using the regional Digital Elevation Map. The daily rate of the chemicals is computed using a database containing the mean values per year in each Council, with a seasonal modulation, in order to include the effects of the crops and the constraints to the chemical and organic spreading issued by Regional Council.

The field spreading has a monthly variability, according to coefficients related to natural contributions (atmospheric deposition and natural production) and artificial contributions (from cattle and pig breeding and chemical fertilization). The artificial contributions are dominant and are strong in spring, for chemicals, and in autumn, for zootechny.

The definition in each cell of the hydrological and sediment variables (governing the quality variables at versant scale) is the output of the hydrologic and the erosion module. The unavailability of direct quality measurements at a cell level forces the indirect calibration of the model using the measurements in the stream. It also suggests the opportunity of using a limited number of parameters. To allow an easy calibration, three zones are selected, wherein the decay coefficient $K_{dec}$ in eqn (1) is uniform. The three zones are the mountain range, the hilly areas and the flat lands. The reduction of the number of degrees of freedom is also related to the hydrological processes and the "mass response function": it is assumed that chemical response to hydraulic inputs decays within a day.

The main parameters for calibration are: A) the maximum storage $Acc_{max}$; B) the exponents in the various balance equations; C) the multipliers of the decay coefficients in each catchment; D) the coefficients parametrizing the transfer from catchment to the main stream.

Table 1 reports some statistical parameters of the time series obtained applying the model to four basins. The period of simulation is from 01.01.1993 to 30.04.1999. The main role is played by generated and decayed quantities for each of the nutrient, but with a varying ratio for each basin. It is also a consequence of the different measurement stations used for calibration and of the uncertainties in assessing the hydrological and chemical forcing terms.

Figure 1 shows the year average value of the mass of $BOD_5$ discharged in the main stream from the catchment basin of Nure River. The plot allows an easy evaluation of the most active area in polluting surface water.

The sediment transport model computes, for each cell, the sediment budget including the contribution (if present) of the surrounding cell. The last cells are tributaries of the main stream. The algorithm has a time step of one day assuming a quasi-steady condition; the parameters were obtained using the thematic maps of the Regione Emilia Romagna. The range of values is [0-0.36] for *K*, the parameter of intrinsic erodibility of the soil, and [0.0001-0.5] for the cropping parameter *C*. The calibration was carried out using data by Cati, [2].

The computed values obtained applying the model are reported in Table 3 in terms of percentiles and related to the closure section of the basin. The dashed values in Table 2 and Table 3 can be compared. No measured data are available for Nure River. Figure 2 shows the erosion/deposit map for one of the basins.

Table 1: Output of the quality model.

| **Balance of BOD5** | | **Nure** | **Taro** | **Secchia** | **Panaro** |
|---|---|---|---|---|---|
| Basin area | ($km^2$) | 463 | 2050 | 2152 | 1776 |
| Mean value generated | (kg/y/ha) | 54.9 | 116.8 | 239.2 | 172.3 |
| Initial | (t) | 127.0 | 1437.0 | 2557.0 | 1529.0 |
| Generated in the field | (t) | 16 075.0 | 15 1382.0 | 323 213.0 | 70 959.0 |
| Final | (t) | 117.0 | 1834.0 | 6920.0 | 1754.0 |
| Emptied in the stream | (t) | 2402.0 | 27 364.0 | 35 327.0 | 3396.0 |
| Infiltrated | (t) | 855.0 | 3329.0 | 2952.0 | 486.0 |
| Decayed | (t) | 12 827.0 | 120 293.0 | 280 571.0 | 66 852.0 |
| **Balance of Nitrogen** | | **Nure** | **Taro** | **Secchia** | **Panaro** |
| Mean value generated | (kg/y/ha) | 68.2 | 93.2 | 154.3 | 131.6 |
| Initial | (t) | 158.0 | 1146.0 | 1650.0 | 1168.0 |
| Generated in the field | (t) | 20 457.0 | 122 085.0 | 210 215.0 | 56 468.0 |
| Final | (t) | 235.0 | 1236.0 | 2083.0 | 1186.0 |
| Emptied in the stream | (t) | 1880.0 | 15 042.0 | 12 391.0 | 2164.0 |
| Infiltrated | (t) | 751.0 | 3548.0 | 2704.0 | 434.0 |
| Decayed | (t) | 17 750.0 | 10 3405.0 | 194 686.0 | 53 853.0 |
| **Balance of Phosphorus** | | **Nure** | **Taro** | **Secchia** | **Panaro** |
| Mean value generated | (kg/y/ha) | 22.6 | 24.9 | 37.7 | 34.2 |
| Initial | (t) | 52.0 | 306.0 | 403.0 | 304.0 |
| Generated in the field | (t) | 6947.0 | 33 404.0 | 52 031.0 | 15 293.0 |
| Final | (t) | 41.0 | 112.0 | 322.0 | 676.0 |
| Emptied in the stream | (t) | 92.0 | 1292.0 | 1315.0 | 520.0 |
| Infiltrated | (t) | 484.0 | 1327.0 | 1750.0 | 569.0 |
| Decayed | (t) | 6381.0 | 30 979.0 | 49 047.0 | 13 831.0 |

A sensitivity test, developed varying the exponent $n$ in the function of detachment, the coefficient of erodibility $K$ and the cropping $C$, the internal friction angle of the soil and the two coefficients $c_1$ and $c_2$ of the average fly distance and of detachment. The time constant was assumed negligible. Table 4 reports the results for the basin of the Nure River (assuming $n_o = 2$).

## 5 Conclusions

The model can be efficiently used whenever a limited number of parameters are known or can be evaluated. The calibration relies on data acquired in the streams, which have to be modelled separately; presently, no data are available for a direct calibration of the surface processes. The sensitivity tests (carried out only for sediment transport) indicate that the exponent of sediment discharge function is the most influencing term.

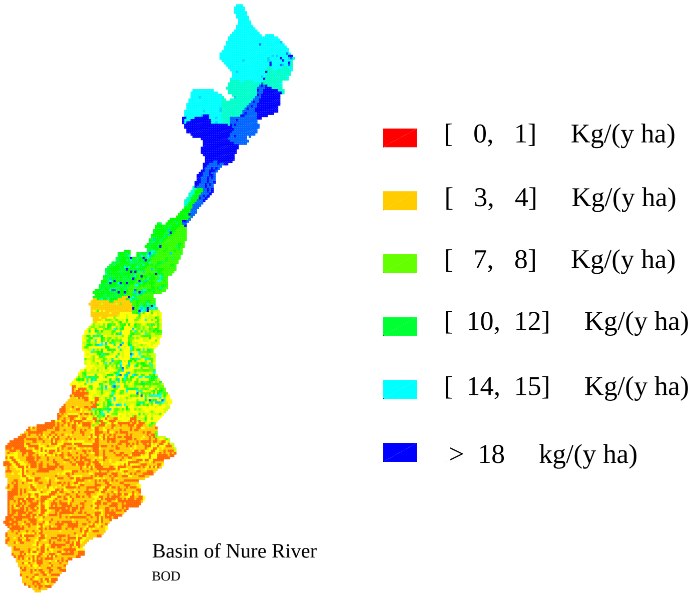


Figure 1: Basin of Nure River. Average year discharge of $BOD_5$ in the main stream.

Table 2: Average year erosion measure for some basins in Emilia Romagna, Italy (by Cati, [2]).

| Stream and Station | years | Basin in km$^2$ | Average elevation above sea level s.m.m | t/km$^2$ | m$^3$/km$^2$ | Average year erosion, in mm |
|---|---|---|---|---|---|---|
| | | | | | | |
| Taro a Piane di Carn. | 7 | 90.5 | 970 | 356 | 130 | **0.013** |
| Taro ad Ostia | 5 | 408 | 824 | 296 | 109 | **0.0109** |
| Taro a S.Quirico | 17 | 1476 | 660 | 1109 | 412 | **0.412** |
| Secchia at Cavola bridge | 5 | 341 | 965 | 1440 | 527 | **0.527** |
| Secchia a Castellarano | 5 | 941 | 831 | 1070 | 393 | **0.393** |
| Secchia at Bacchello bridge | 22 | 1292 | 606 | 1847 | 684 | **0.684** |
| Panaro at Samone bridge | 5 | 589 | 824 | 1610 | 594 | **0.594** |
| Panaro a Comporto | 19 | 1036 | 662 | 2030 | 769 | **0.769** |

Table 3: Statistical indicators of the output of the sediment transport model.

| (Erosion in mm per year) | Nure | Taro | Secchia | Panaro |
|---|---|---|---|---|
| Erosion 1st quartile | 0.00 | 0.01 | 0.00 | 0.01 |
| Erosion median | 0.03 | 0.06 | 0.03 | 0.09 |
| Erosion 3rd quartile | 0.82 | 0.43 | 0.74 | 0.84 |

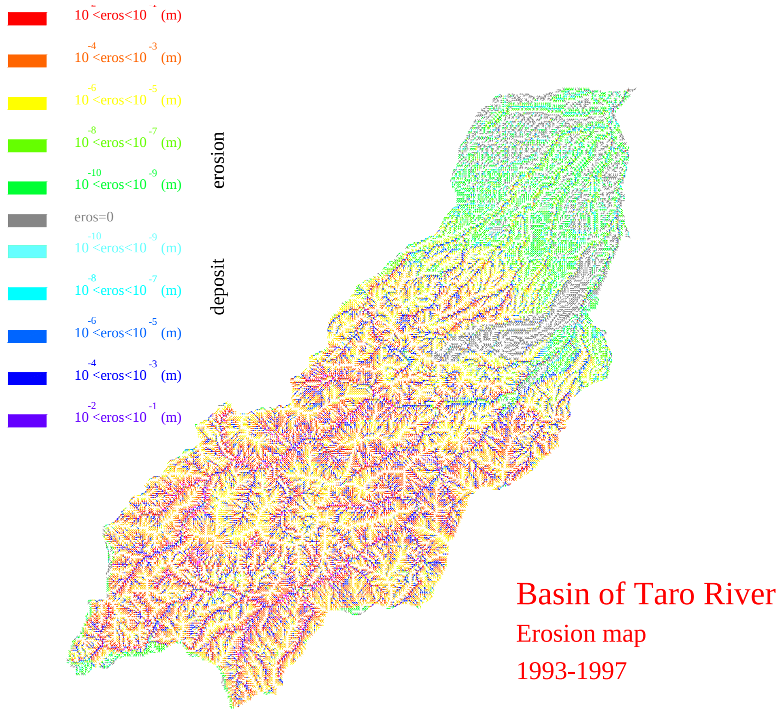


Figure 2: Basin of Taro River: erosion map.

Table 4: Sensitivity analysis for the basin of the Nure River.

| | $n_o = 2$ | $n = 1.1n_o$ | $n = 0.9n_o$ | $K$=1.1$K_o$ | $K$=0.9$K_o$ | $C$=1.1$C_o$ | $C$=0.9$C_o$ |
|---|---|---|---|---|---|---|---|
| Median | 1 | +33% | -28% | +1.2% | -5.1% | +.9% | -4.0% |
| 3rd quartile | 1 | +69.5% | -36.5% | 9.7% | -10.9% | +8.9% | -9.1% |

| | $n_o$ =2 | $c_1$=1.1 $c_{1o}$ | $c_1$=0.9 $c_{1o}$ | $c_2$=1.1 $c_{2o}$ | $c_2$=0.9 $c_{2o}$ | $\phi$=1.1$\phi$o | $\phi$=0.9$\phi$o |
|---|---|---|---|---|---|---|---|
| Median | 1 | +1.9% | -3.7% | +1.6% | -3.1% | +0.6% | -0.1% |
| 3rd quartile | 1 | +9.2% | -9.1% | +9.1% | -7.5% | +3.1% | -2.5% |

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