Supplementary material for Radial gravity currents in vertically graded porous media: theory and experiments for Newtonian and power-law fluids

Vittorio Di Federico^a, Sandro Longo^b, Luca Chiapponi^b, Renata Archetti^a, Valentina Ciriello^a

 ^aDipartimento di Ingegneria Civile, Chimica, Ambientale e dei Materiali (DICAM), Università di Bologna, Viale Risorgimento, 2, 40136 Bologna, Italy
^bDipartimento di Ingegneria Civile, Ambiente Territorio e Architettura (DICATeA), Università di Parma, Parco Area delle Scienze, 181/A, 43124 Parma, Italy

1. The uncertainty analysis

In order to verify the ability of the model to reproduce the experimental results in various conditions, the same uncertainty analysis reported in the main manuscript only for test 13 has been extended here to all tests. Figures S.1-S.11 present the experimental data, the theoretical results and the error bands (95% confidence limits) for the tests listed in table 1, at a time correspondent to the last available shot.



Figure S.1: Test 14, $m = 0.60 \pm 3.5\%$ Pa·sⁿ, $n = 0.33 \pm 3.5\%$, $\Delta \rho = 1175 \pm 1.0\%$ kg·m⁻³, $Q' = 0.078t^{1/2} \pm 0.5\%$ ml·s⁻¹, $\alpha = 1.5$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 3.47 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.2: Test 15, $m = 0.60 \pm 3.5\%$ Pa·sⁿ, $n = 0.33 \pm 3.5\%$, $\Delta \rho = 1175 \pm 1.0\%$ kg·m⁻³, $Q' = 0.032t \pm 0.5\%$ ml·s⁻¹, $\alpha = 2.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 4.57 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.3: Test 19, $m = 0.28 \pm 3.5\%$ Pa·sⁿ, $n = 1.0 \pm 3.5\%$, $\Delta \rho = 1241 \pm 1.0\%$ kg·m⁻³, $Q' = 0.40 \pm 0.5\%$ ml·s⁻¹, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 9.63 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.4: Test 20, $m = 0.28 \pm 3.5\% \text{ Pa} \cdot \text{s}^n$, $n = 1.0 \pm 3.5\%$, $\Delta \rho = 1241 \pm 1.0\% \text{ kg} \cdot \text{m}^{-3}$, $Q' = 0.06t^{1/2} \pm 0.5\% \text{ ml} \cdot \text{s}^{-1}$, $\alpha = 1.5$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 8.14 \cdot 10^{-9} \pm 4.8\% \text{ m}^2$. The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.5: Test 21, $m = 0.28 \pm 3.5\% \text{Pa} \cdot \text{s}^n$, $n = 1.0 \pm 3.5\%$, $\Delta \rho = 1241 \pm 1.0\% \text{kg} \cdot \text{m}^{-3}$, $Q' = 0.032t \pm 0.5\%$ ml·s⁻¹, $\alpha = 2.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 9.87 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.6: Test 22, $m = 0.67 \pm 3.5\%$ Pa·sⁿ, $n = 0.42 \pm 3.5\%$, $\Delta \rho = 1175 \pm 1.0\%$ kg·m⁻³, $Q' = 0.550 \pm 1.0\%$ ml·s⁻¹, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 8.97 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.7: Test 23, $m = 0.67 \pm 3.5\%$ Pa·sⁿ, $n = 0.42 \pm 3.5\%$, $\Delta \rho = 1175 \pm 1.0\%$ kg·m⁻³, $Q' = 0.677 \pm 1.0\%$ ml·s⁻¹, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 9.37 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.8: Test 24, $m = 0.67 \pm 3.5\%$ Pa·sⁿ, $n = 0.42 \pm 3.5\%$, $\Delta \rho = 1175 \pm 1.0\%$ kg·m⁻³, $Q' = 0.866 \pm 1.0\%$ ml·s⁻¹, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 9.85 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.9: Test 25, $m = 0.14 \pm 3.5\% \text{Pa} \cdot \text{s}^n$, $n = 1.0 \pm 3.5\%$, $\Delta \rho = 1241 \pm 1.0\% \text{kg} \cdot \text{m}^{-3}$, $Q' = 0.535 \pm 1.0\% \text{ ml} \cdot \text{s}^{-1}$, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 8.43 \cdot 10^{-9} \pm 4.8\% \text{m}^2$. The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.10: Test 26, $m = 0.14 \pm 3.5\%$ Pa·sⁿ, $n = 1.0 \pm 3.5\%$, $\Delta \rho = 1241 \pm 1.0\%$ kg·m⁻³, $Q' = 0.704 \pm 1.0\%$ ml·s⁻¹, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 9.00 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.



Figure S.11: Test 27, $m = 0.14 \pm 3.5\%$ Pa·sⁿ, $n = 1.0 \pm 3.5\%$, $\Delta \rho = 1241 \pm 1.0\%$ kg·m⁻³, $Q' = 0.888 \pm 1.0\%$ ml·s⁻¹, $\alpha = 1.0$, $\phi = 0.37 \pm 1\%$, $\omega = 1.63 \pm 4.3\%$, $k_0 = 9.51 \cdot 10^{-9} \pm 4.8\%$ m². The error bar and the confidence limits (dashed red curves) refer to 95% confidence level.