Rayleigh-Bénard instability of the power-law fluid flow in a porous medium: numerical and experimental analysis

M. Celli[⊛] A. Barletta[⊛] S. Longo[†] V. Di Federico[⊛] L. Chiapponi[†] A. Valiani[‡]

[⊛] Università di Bologna - Italy [†] Università di Parma - Italy [‡] Università di Ferrara - Italy

Theoretical Analysis

The linear stability of fluid a saturated porous channel heated from below is studied. The governing equations are the following

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\mu^*(T)}{K}\,|\mathbf{u}|^{n-1}\mathbf{u} = -\nabla p - \rho_0\,\mathbf{g}\,\beta\,(T - T_0)$$

$$\sigma\,\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \varkappa \nabla^2 T$$

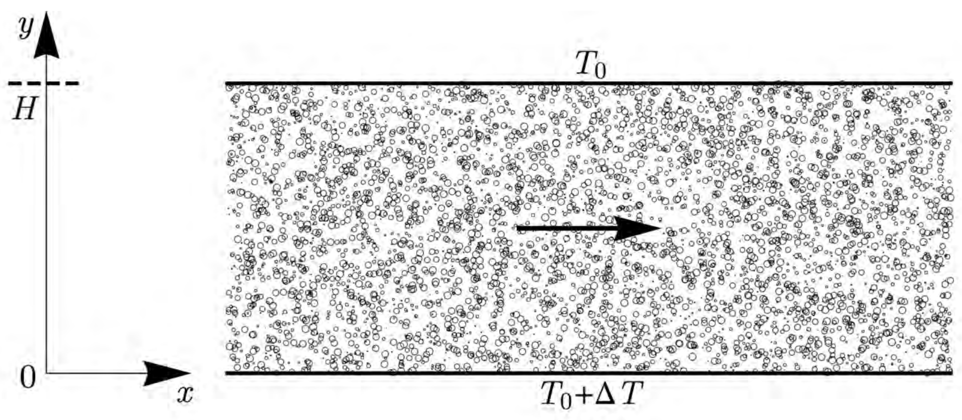
$$y = 0: \qquad v = 0, \quad T = T_0 + \Delta T$$

$$y = H: \qquad v = 0, \quad T = T_0$$

The consistency index of the power law fluid depends on temperature

$$\mu^*(T) = \mu_0^*\,[1 + \xi\,(T - T_0)]^{-n}$$

Where ξ is a fluid property modulating the slope of the temperature change of the consistency index.

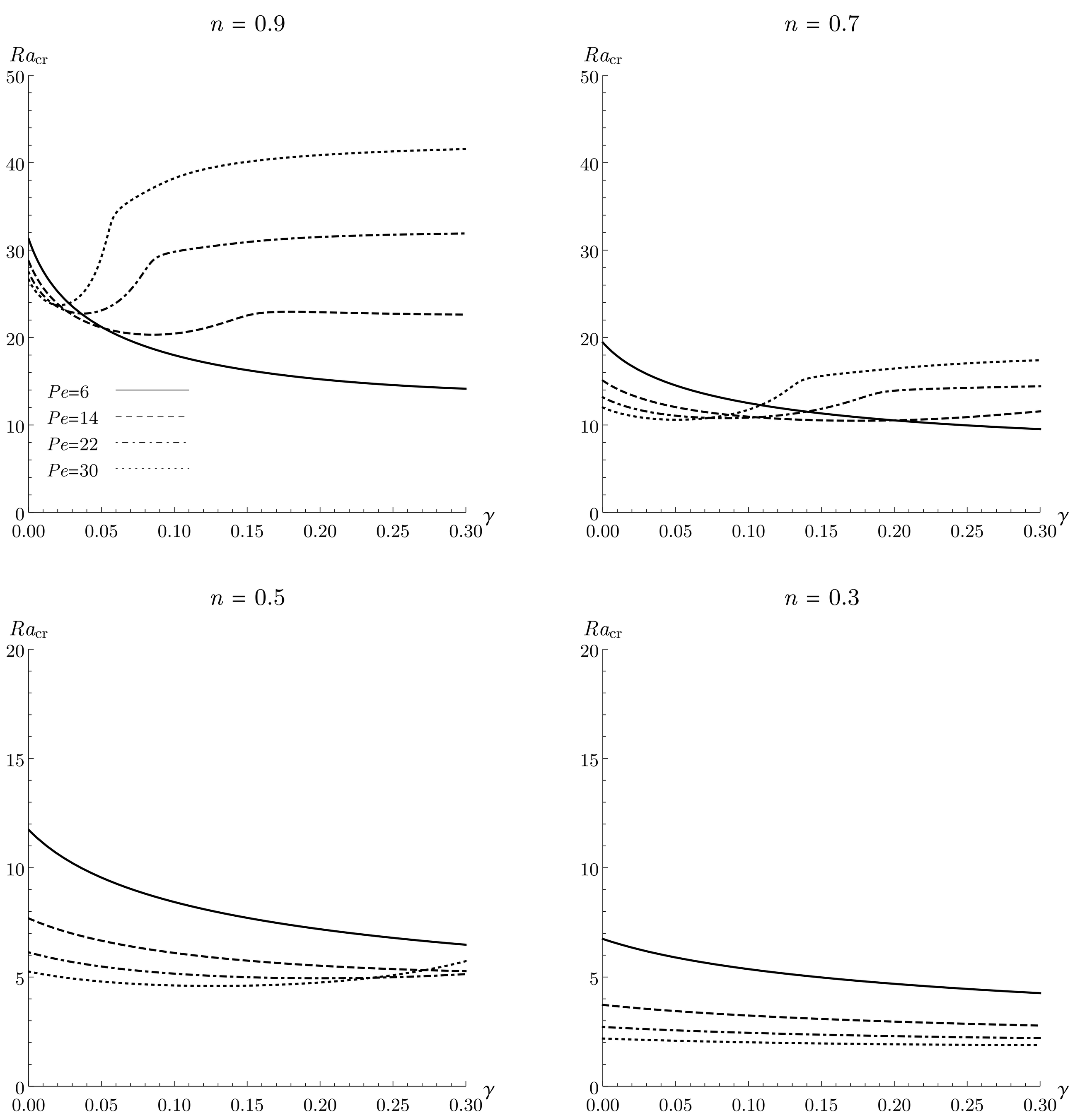


Geometry of the fluid a saturated porous channel

A two dimensional, (x, y), configuration is studied.

RESULTS

The stability analysis consists in finding the critical values that identify the threshold conditions for the onset of convective instability.



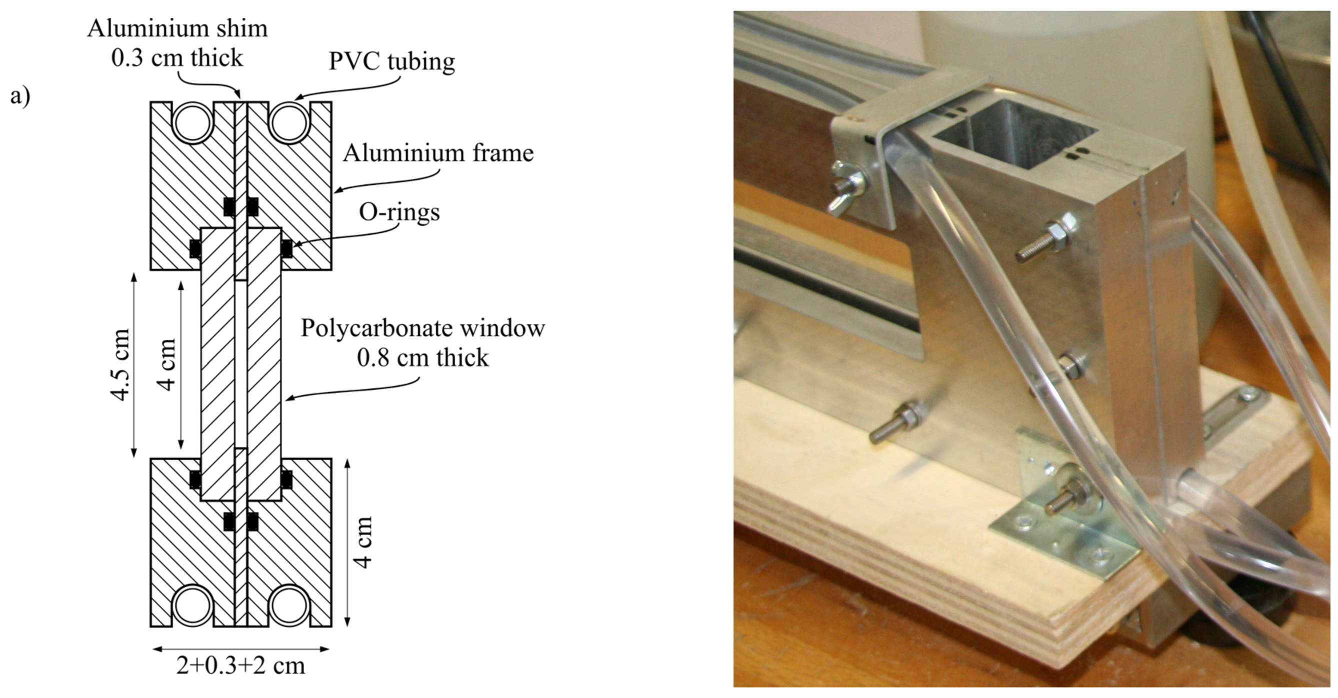
Critical values of the governing parameter Ra

Where the governing parameter here are the Rayleigh number Ra, γ a non-negative dimensionless parameter that tunes the departure from the constant consistency index model and the Péclet number Pe

$$Ra = \frac{\rho_0\,g\,\beta\,\Delta T\,K\,H^n}{\mu_0^*\,\varkappa^n} \qquad \gamma = \frac{\mu_0^*\,\varkappa^n\,\xi}{\rho_0\,g\,\beta\,K\,H^n} \qquad Pe = \frac{u_0\,H}{\varkappa}$$

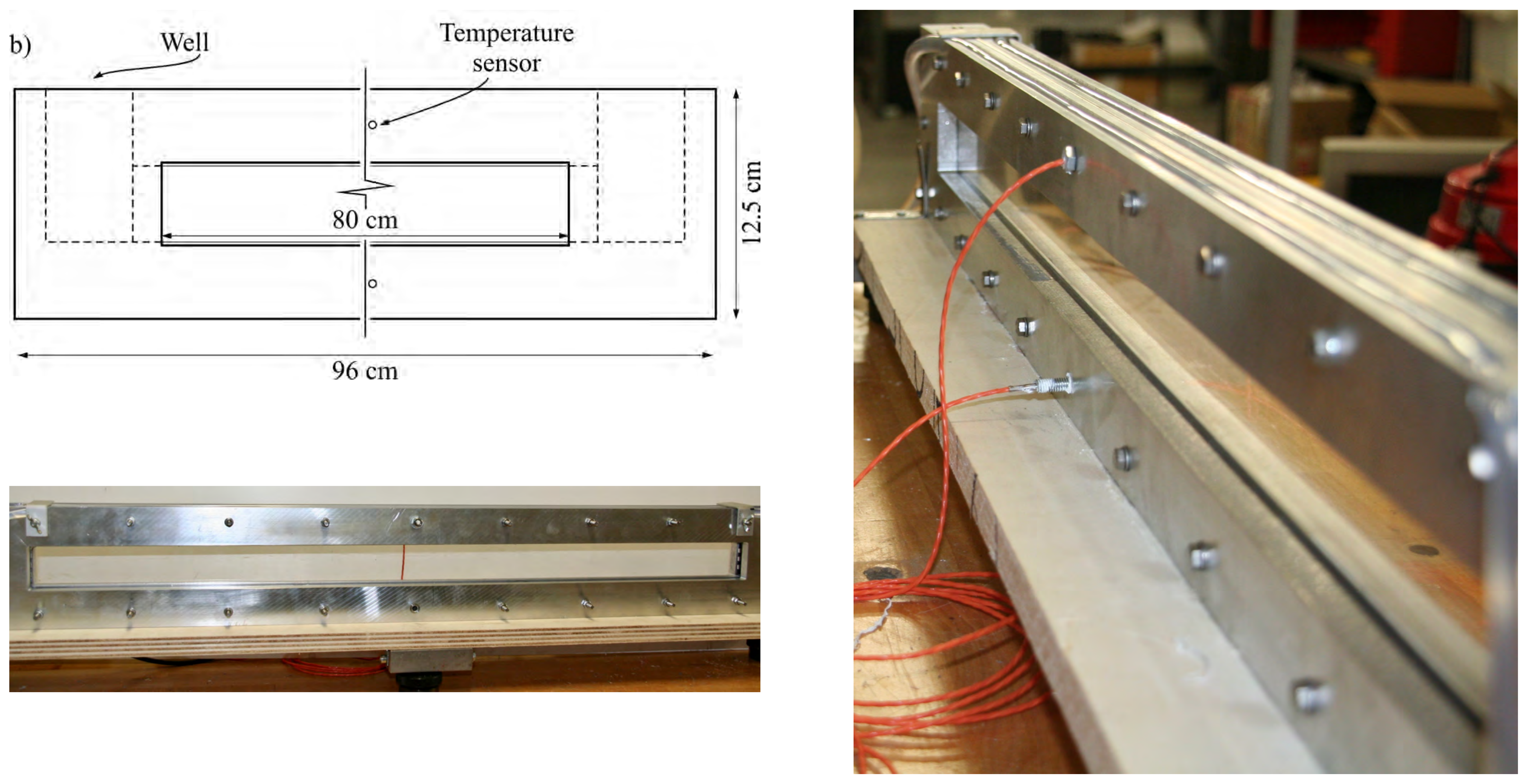
Experimental Analysis

The two dimensional behaviour of the fluid saturated porous medium is simulated experimentally by employing a Hele-Shaw cell of length $L = 80$ cm and height $H = 4$ cm similar to Hartline and Lister (1977)



Cross-section of the Hele-Shaw cell

The optical access is through two polycarbonate windows 0.8 cm thick, with a gap maintained by aluminium shims of 0.1, 0.2, and 0.3 cm. The assembly is held together by an aluminium frame machined by using a CNC milling cutter. The temperature control is obtained by circulating water at the hot lower side, and coolant at the cold upper side.

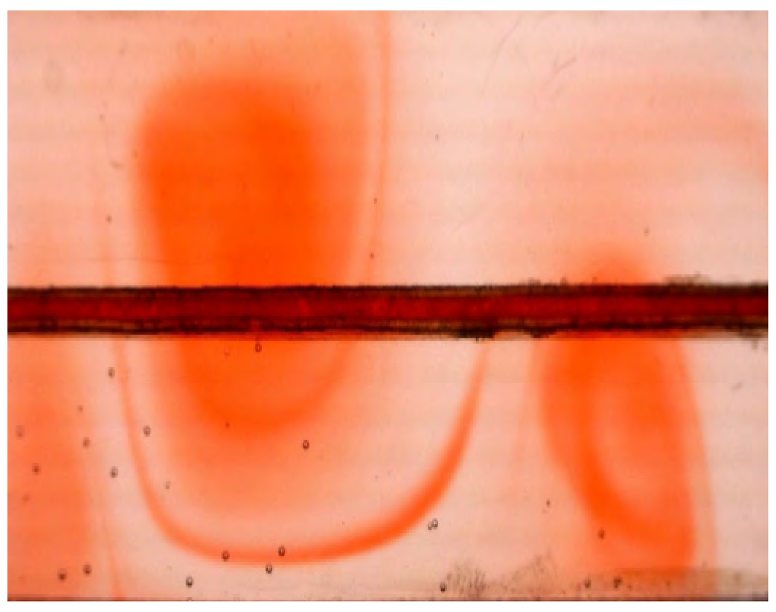


Front and view of the Hele-Shaw cell

The rheology of the fluid was measured with a parallel plate rheometer (Dynamic Shear Rheometer Anton Paar Physica MCR 101), and the mass density with a hydrometer (STV3500/23 Salmoiraghi), with an accuracy of 1%.

PRELIMINARY RESULTS

The flow visualization is obtained by injecting a colored ink inside the cell. This method does not affect the flow features.



Temperature difference $\Delta T = 16.3$ K

Temperature difference $\Delta T = 10.4$ K

Preliminary results show that the convective instability arises for values of Rayleigh number compatible with those obtained by the numerical analysis.

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